

International Journal of Modern Physics A
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ACCELERATING COSMOLOGIES AND INFLATION FROM M/SUPERSTRING THEORIES

NOBUYOSHI OHTA

Department of Physics, Osaka University, Toyonaka, Osaka 560-0043, Japan
ohta@phys.sci.osaka-u.ac.jp

Preprint OU-HET 500
 Invited review to be published in IJMPA

We review the recent developments in obtaining accelerating cosmologies and/or inflation from higher-dimensional gravitational theories, in particular superstring theories in ten dimensions and M-theory in eleven dimensions. We first discuss why it is difficult to obtain inflationary behavior in the effective low-energy theories of superstring/M-theory, i. e. supergravity theories. We then summarize interesting solutions including S-branes that give rise to accelerating cosmologies and inflationary solutions in M-theory with higher order corrections. Other approaches to inflation in the string context are also briefly discussed.

Keywords: Accelerating Cosmologies; Inflation; M/Superstring theories.

1. Introduction

Our universe is believed to have undergone an inflationary evolution in the early epoch. Why is this so? The reason is that this mechanism naturally explains the following major questions in the current cosmology:^{1,2}

- Horizon problem: Why is the early universe so highly homogeneous beyond causally connected regions?
- Flatness problem: Why does the present universe appear so extremely flat?

In fact the recent cosmological observation seems to confirm its existence.³ Not only that, it is discovered that the present universe still expands with acceleration (late-time acceleration). It is not difficult to construct cosmological models with these features in the Einstein gravity by suitable modifications. For example, if one introduces a positive cosmological constant, it is possible to get de Sitter expansion of exponential type. A scalar field with positive potential can also be used for this purpose. Also it has been pointed out that higher order curvature terms can do similar job.⁴

However it is desirable to derive such a model from the fundamental theories of particle physics that incorporate gravity without making special assumptions in the theories. Any theory that claims to be the correct theory of gravity must explain

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these accelerating cosmologies as well as inflation. The most plausible candidate for the consistent theory of quantum gravity is superstrings or M-theory. So it is very important to see how one can get a good model from these theories.

When one tries to consider this problem in string theory, one is faced with various problems. First of all, string theories on time-dependent backgrounds are notoriously difficult to deal with. Attempts in this direction are made in Refs. 5 and 6 using the simplest orbifolds, and cosmological implications are studied. Other approaches to strings on time-dependent backgrounds include Refs. 7, 8. This subject is reviewed in Ref. 9. However, it is rather difficult to discuss other time-dependent backgrounds in the context of quantum string theories. So it is more practical to approach the problem using the low-energy effective supergravities.

Even within the supergravities, as we shall first summarize in the next section, there is a no-go theorem that de Sitter solutions with accelerating expansion are not obtained at the stationary minima in superstring or supergravity theories. We must somehow try to evade this theorem by relaxing some assumptions involved in the proof of the theorem. Several attempts in this direction are well reviewed in Refs. 10 and 11, and the present paper also summarizes various proposals, focusing mainly on time-dependent solutions to the (generalized) Einstein equations in higher-dimensional supergravities.^{12,13,14,15,16}

In the string theory, there are important extended objects called branes. Many of the proposals to derive inflation in the string context make use of these extended objects. In this approach, one considers that our world exists on such branes. We shall not discuss this subject much since such attempts to derive inflation or alternatives to inflation from the dynamics of branes are nicely summarized in Ref. 17.

This review is organized as follows. In the next section, we start with summarizing the no-go theorem that claims that de Sitter solutions are impossible in supergravity theories, and assumptions involved in the proof. This clarifies what kind of approaches enable us to evade the no-go theorem.

In Sec. 3, we first show that relaxing one of the conditions in the proof of the no-go theorem, we can get time-dependent solutions to vacuum Einstein equation in higher dimensions that give accelerating cosmology.

In Sec. 4, we show that this class of solutions are actually special cases of what are known as S-brane solutions in supergravity. We first summarize the solutions in Sec. 4.1, discuss the relation to the vacuum solution in Sec. 4.2, study cosmological aspects of the S-brane solutions in Sec. 4.3 and show that one cannot in general obtain e-folding large enough to solve the cosmological problems mentioned above. We also discuss related solutions in type II superstrings in Sec. 4.4, and give intuitive understanding of the basic mechanism of the accelerating behavior of the solutions and why it is difficult to obtain large e-folding number in these models in Sec. 4.5. This leads us to search for other solutions with larger e-foldings.

In Sec. 5, we discuss an attempt to obtain larger e-foldings in this context, with eternal expansion by considering hyperbolic spaces both for external and internal spaces. This class of solutions may be useful as a models for the present accelerating

cosmology.

The generic feature of the higher dimensional gravitational theories is that they give scalar field theories coupled to gravity in the four-dimensional point of view. In Sec. 6, we summarize cosmological solutions in the theories with single scalar field with exponential potential and classify possible solutions. The case of multi-scalars is much more difficult and we only refer to references discussing various theories of this type.

In Sec. 7, we consider another approach that avoids the no-go theorem by considering higher order corrections to the low-energy effective theories of M-theory. The corrections are given as a special combination of R^4 terms of Lovelock type (eight-dimensional Euler density) and specific terms containing higher derivatives. The basic equations for the theory are relegated to Appendix. We discuss solutions in the system in Sec. 7.1, study their stability in Sec. 7.2 and give possible scenario that can be obtained within these solutions in Sec. 7.3.

In Sec. 8, other approaches to accelerating universe are briefly summarized.

Finally in Sec. 9, we conclude with the summary of the results reviewed in this paper and outlook.

2. No-go theorem

The Einstein equation provide the following equations:

$$\text{Raychaudhuri eq.: } \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P), \quad (1)$$

$$\text{Friedmann eq.: } \left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho, \quad (2)$$

for the energy density ρ and pressure P . The weak energy condition requires that

$$\rho \geq 0. \quad (3)$$

In order to get inflation $\ddot{a} > 0$, $\rho + 3P$ must be negative ($w \equiv \frac{P}{\rho} < -\frac{1}{3}$), which means anti-gravitation or gravitational repulsion. This sounds physically nonsensical, but this is possible under some circumstances. A famous example is the positive cosmological constant, which gives

$$\rho = \Lambda, \quad P = -\Lambda, \quad (4)$$

so that indeed we have $\rho + 3P = -2\Lambda < 0$, giving repulsion. Since we have

$$\rho + 3P = 2(T_{00} - \frac{1}{2}g_{00}T^\lambda_\lambda) = \frac{1}{4\pi G}R_{00}, \quad (5)$$

Raychaudhuri equation reduces to

$$\frac{\ddot{a}}{a} = -\frac{1}{3}R_{00}. \quad (6)$$

So in order to have inflation, we must have $R_{00} < 0$, i. e. *strong energy condition must be violated*.¹⁸

However it is not violated by eleven-dimensional supergravities or any ten-dimensional supergravity theories corresponding to the low-energy effective theories for M/superstring theories. Moreover, if the higher-dimensional stress tensor satisfies the strong energy condition, then so does the lower-dimensional stress tensor. All of this implies that solutions of accelerated expansion are impossible in supergravities as the low-energy effective theories of M/superstrings. Another and more detailed argument for this no-go theorem is given below.

2.1. Assumptions

We consider $D(> 2)$ -dimensional gravity, which is compactified on d dimensions.¹⁹ D -dimensional indices are denoted by M, N, L, \dots whereas d -dimensional ones by μ, ν, ρ, \dots and the rest by m, n, l, \dots

We assume the following:

- (1) Gravitational interactions involve no higher derivative terms.
- (2) Potential is not positive definite.
- (3) All the massless fields in the theory have positive kinetic terms.
- (4) d -dimensional Newton constant is finite.

These are the assumptions in proving the no-go theorem. The second point may appear strange, so let us explain why this is natural. Take the metric of our D -dimensional space as

$$ds^2 = e^{-\frac{2(D-d)}{d-2}\phi} ds_d^2 + e^{2\phi} d\Sigma_{D-d,\sigma}^2, \quad (7)$$

where σ denotes the sign of the curvature of the internal space. This is chosen such that the resulting d -dimensional theory involves the ordinary Einstein term without multiplicative factors. This is known as the Einstein frame in d dimensions. Substituting (7) into the Einstein action, we get the effective potential in d dimensions (see Sec. 4.5):

$$V = -\sigma \frac{(D-d)(D-d-1)}{2} \exp \left[-\frac{2(D-2)}{d-2} \phi \right]. \quad (8)$$

Usually the internal space is chosen to be compact space with positive curvature $\sigma = +1$, which means that the potential is negative. The potential minimum is 0 with the field value at infinity for $\epsilon = -1$ and for any value of field for $\epsilon = 0$. With the presence of matter fields, the minima always take zero or negative values, giving Minkowski or anti-de Sitter spaces. This is probably closely related to the fact that supersymmetry is possible only in flat Minkowski or anti-de Sitter spaces but not in de Sitter space.

2.2. Proof

We write D -dimensional Einstein equation:

$$R_{MN} = T_{MN} - \frac{1}{D-2} g_{MN} T^L_L, \quad (9)$$

and the metric

$$ds^2 = \Omega^2(y)(dx_d^2 + \hat{g}_{mn}dy^m dy^n), \quad (10)$$

where the first line element $dx_d^2 = \xi_{\mu\nu} dx^\mu dx^\nu$ denotes Minkowski or de Sitter space. From the Einstein equation (9), we have

$$\begin{aligned} R_{\mu\nu} &= R_{\mu\nu}(\xi) - \xi_{\mu\nu}(\hat{\nabla}^2 \log \Omega + (D-2)(\hat{\nabla} \log \Omega)^2) \\ &= T_{\mu\nu} - \frac{1}{D-2}\Omega^2 \xi_{\mu\nu} T^L_L, \end{aligned} \quad (11)$$

where the hat means that the contraction is made by \hat{g}_{mn} . Contracted with ξ , Eq. (11) gives

$$R(\xi) - d(\hat{\nabla}^2 \log \Omega + (D-2)(\hat{\nabla} \log \Omega)^2) = \left(T^\mu_\mu - \frac{d}{D-2} T^L_L \right) \Omega^2, \quad (12)$$

and hence

$$\hat{\nabla}^2 \log \Omega + (D-2)(\hat{\nabla} \log \Omega)^2 = \frac{1}{d} \left[R(\xi) + \Omega^2 \left(-T^\mu_\mu + \frac{d}{D-2} T^L_L \right) \right], \quad (13)$$

where the energy-momentum tensors on the rhs are contracted with the D -dimensional metric. If we define

$$\tilde{T} \equiv -T^\mu_\mu + \frac{d}{D-2} T^L_L, \quad (14)$$

this is non-negative. This can be verified as follows. If we have potential V for matter fields, we have $T_{MN} \sim -V g_{MN}$ and so

$$\tilde{T} = Vd - \frac{d}{D-2} DV = -\frac{2d}{D-2} V \geq 0, \quad (15)$$

where the last equality follows from our assumption (2). If we have n -form fields, the energy momentum tensor takes the form

$$T_{MN} = F_{ML_1 \dots L_{n-1}} F_N^{L_1 \dots L_{n-1}} - \frac{1}{2n} g_{MN} F^2, \quad (16)$$

which gives $T^\mu_\mu = F_{\mu L_1 \dots L_{n-1}} F^{\mu L_1 \dots L_{n-1}} - \frac{d}{2n} F^2$. Hence

$$\tilde{T} = -F_{\mu L_1 \dots L_{n-1}} F^{\mu L_1 \dots L_{n-1}} + \frac{d}{D-2} \left(1 - \frac{1}{n} \right) F^2. \quad (17)$$

Here all the indices to F belong to the internal space, or if $n \geq d$ to all the d dimensions and part of internal space (otherwise the isometry of R^d or dS^d is broken). They separately contribute to \tilde{T} . In the former case, $F^2 \geq 0$. It follows from (17) that $\tilde{T} \geq 0$ for $n > 1$ and $\tilde{T} = 0$ for $n = 1$. In the latter case, $F^2 < 0$ and $F_{\mu L_1 \dots L_{n-1}} F^{\mu L_1 \dots L_{n-1}} = \frac{d}{n} F^2$. It again follows

$$\begin{aligned} \tilde{T} &= \left[-\frac{d}{n} + \frac{d}{D-2} \left(1 - \frac{1}{n} \right) \right] F^2 \\ &= -\frac{d(D-2-n+1)}{n(D-2)} F^2 \geq 0. \end{aligned} \quad (18)$$

Consequently we have in general

$$\tilde{T} \geq 0. \quad (19)$$

Combined with Eq. (13) and the assumption that our d -dimensional space is Minkowski or de Sitter with non-negative scalar curvature, this means that

$$\Omega^{D-2} \hat{\nabla}^2 \Omega^{D-2} \geq 0. \quad (20)$$

The equality is true only for Minkowski space. Integrating this over our internal space, we get

$$\int d^{D-d}y \sqrt{-\hat{g}} (\hat{\nabla} \Omega^{D-2})^2 \leq 0, \quad (21)$$

where we have made partial integration. The lhs is a positive-semi-definite quantity so that this is valid only if Ω is constant and the equality holds. This implies that the rhs of (13) vanishes and hence de Sitter space is not allowed since the second term is positive. Note that at this last step, we have assumed that there is no contribution from surface terms upon partial integration. If the manifold has (singular) boundary which produces surface contributions, the no-go theorem breaks down.

2.3. How to avoid the theorem

Given the confirmation of the accelerated expansion of our universe both at early time and present,³ we have to find out how this no-go theorem may be evaded. This can be done if various assumptions, implicit and explicit, are relaxed. Here are some possibilities.

- It is assumed that the size of the internal space is time-independent. We can try to introduce the time-dependence of the internal space.
- We can consider higher-derivative “quantum correction” terms which are known to exist in M-theory and superstrings.
- We give up the compact and smooth condition on the internal space without boundary.

The first possibility leads to the recent progress using time-dependent solutions, and this approach is discussed in detail in this paper. The result is that interesting solutions are obtained, but the expansion is not enough for the inflation at early universe. However it may be useful for the present accelerating expansion though there remain some problems to be overcome to achieve this.

Alternatively, we can consider higher-derivative “quantum correction” terms which are known to exist in M-theory and superstrings. We shall show that indeed this possibility leads to a very promising direction of achieving inflation.

If, however, we accept supergravities without such higher order corrections, then we must give up the compact and/or no-boundary condition on the internal space, but this would lead to continuous spectrum in four dimensions, which is not desirable. Compact space with boundary is likely to suffer from nonresolvable singularities. This possibility is not explored in this paper.

3. Accelerating Cosmology from Vacuum Einstein Equation

The first example of accelerating cosmology in the context of higher-dimensional (super)gravity was obtained simply as a solution to the vacuum Einstein equation but with hyperbolic internal space.¹² (The use and relevance of the compact hyperbolic manifolds for the internal space was first noted in Ref. 20. Similar time-dependent solutions for flat internal space were discussed earlier in Ref. 21.)

It is convenient to write the $(4+n)$ -dimensional solution as

$$ds^2 = \delta^{-n}(t)ds_E^2 + \delta^2(t)d\Sigma_{n,\sigma}^2, \quad (22)$$

where n is the dimension of the internal spherical ($\sigma = +1$), flat ($\sigma = 0$) or hyperbolic ($\sigma = -1$) spaces, whose line elements are $d\Sigma_{n,\sigma}^2$, and

$$ds_E^2 = -S^6(t)dt^2 + S^2(t)d\mathbf{x}^2, \quad (23)$$

describes the 4-dimensional spacetime. The form of (22) is chosen such that the metrics in (23) are in the Einstein frame. The solution is given as

$$\begin{aligned} \delta(t) &= e^{-3t/(n-1)} \left(\frac{\sqrt{3(n+2)/n}}{(n-1)\sinh(\sqrt{3(n+2)/n}|t|)} \right)^{\frac{1}{(n-1)}}, \\ S(t) &= e^{-(n+2)t/2(n-1)} \left(\frac{\sqrt{3(n+2)/n}}{(n-1)\sinh(\sqrt{3(n+2)/n}|t|)} \right)^{\frac{n}{2(n-1)}}. \end{aligned} \quad (24)$$

with hyperbolic internal space.

If we take the time coordinate η defined by

$$d\eta = S^3(t)dt, \quad (25)$$

the metric (23) describes a flat homogeneous isotropic universe with scale factor $S(t)$, and $\delta(t)$ gives the measure of the size of internal space. The condition for expanding 4-dimensional universe is that

$$\frac{dS}{d\eta} > 0. \quad (26)$$

Accelerated expansion is obtained if, in addition,

$$\frac{d^2S}{d\eta^2} > 0. \quad (27)$$

It has been shown that these can be satisfied for $n = 7$ and for certain period of negative t (with the convention $t_1 = 0$) which is the period that our universe is evolving ($t < 0$ and $t > 0$ are two disjoint possible universes).¹² We note that in these solutions, accelerated expansion is obtained only for the hyperbolic internal space; this property is lost for flat or compact internal spaces. We shall show that this problem is circumvented by solutions with flux, known as S-brane solutions.

4. Accelerated Cosmologies from S-branes

The solution in the previous section can be obtained from the S-brane solutions in supergravities.¹³ S-branes are time-dependent solutions in supergravities, originally considered in connection with tachyon condensation and decay.^{22,23,24,25,26,27,28}

4.1. S-branes in supergravities

Let us consider the D -dimensional gravity coupled to a dilaton ϕ and m different n_A -form field strengths:

$$I = \frac{1}{16\pi G_D} \int d^D x \sqrt{-g} \left[R - \frac{1}{2}(\partial\phi)^2 - \sum_{A=1}^m \frac{1}{2n_A!} e^{a_A\phi} F_{n_A}^2 \right]. \quad (28)$$

This action describes the bosonic part of $D = 11$ or $D = 10$ supergravities; we simply drop ϕ and put $a_A = 0$ and $n_A = 4$ for $D = 11$, whereas we set $a_A = -1$ for the NS-NS 3-form and $a_A = \frac{1}{2}(5 - n_A)$ for forms coming from the R-R sector. The field strength for an electrically charged Sq-brane is given by

$$F_{t\alpha_1 \dots \alpha_{q+1}} = \epsilon_{\alpha_1 \dots \alpha_{q+1}} \dot{E}, \quad (n_A = q + 2), \quad (29)$$

where $\alpha_1, \dots, \alpha_{q+1}$ stand for the tangential directions to the Sq-brane. The magnetic case is given by

$$F^{\alpha_{q+2} \dots \alpha_p a_1 \dots a_n} = \frac{1}{\sqrt{-g}} e^{-a\phi} \epsilon^{t\alpha_{q+2} \dots \alpha_p a_1 \dots a_n} \dot{E}, \quad (n_A = D - q - 2) \quad (30)$$

where a_1, \dots, a_n denote the coordinates of the n -dimensional hypersurface $\Sigma_{n,\sigma}$.

In Ref. 23 a single S-brane solution was given, and in Ref. 25 general orthogonally intersecting solutions were derived by solving field equations. Solutions restricted to a single Sq-brane with $(q + 1)$ -dimensional world-volume in p -dimensional space are (hereafter the subscript A is not necessary and is dropped)

$$ds_D^2 = [\cosh \tilde{c}(t - t_2)]^{2\frac{q+1}{\Delta}} \left[e^{2c_0 t + 2c'_0} \left\{ -e^{2ng(t)} dt^2 + e^{2g(t)} d\Sigma_{n,\sigma}^2 \right\} + \sum_{\alpha=1}^p [\cosh \tilde{c}(t - t_2)]^{-2\frac{\gamma(\alpha)}{\Delta}} e^{2\tilde{c}_\alpha t + 2c'_\alpha} (dx^\alpha)^2 \right], \quad (31)$$

$$E = \sqrt{\frac{2(D-2)}{\Delta}} \frac{e^{\tilde{c}(t-t_2)-\epsilon ac'_\phi/2+\sum_{\alpha \in q} c'_\alpha}}{\cosh \tilde{c}(t - t_2)}, \quad \tilde{c} = \sum_{\alpha \in q} c_\alpha - \frac{1}{2} c_\phi \epsilon a, \\ \phi = \frac{(D-2)\epsilon a}{\Delta} \ln \cosh \tilde{c}(t - t_2) + \tilde{c}_\phi t + c'_\phi, \quad (32)$$

where $D = p + n + 1$ and $\epsilon = +1(-1)$ corresponds to electric (magnetic) fields. The coordinates x^α , ($\alpha = 1, \dots, p$) parametrize the p -dimensional space, within which $(q+1)$ -dimensional world-volume of Sq-brane is embedded, and the remaining coordinates of the D -dimensional spacetime are the time t and coordinates on

compact n -dimensional spherical ($\sigma = +1$), flat ($\sigma = 0$) or hyperbolic ($\sigma = -1$) spaces. We have also defined

$$\Delta = (q+1)(D-q-3) + \frac{1}{2}a^2(D-2),$$

$$\gamma^{(\alpha)} = \begin{cases} D-2 & \text{for } \begin{cases} x_\alpha \text{ belonging to } q\text{-brane} \\ \text{otherwise} \end{cases}, \\ 0 & \end{cases} \quad (33)$$

and

$$g(t) = \begin{cases} \frac{1}{n-1} \ln \frac{\beta}{\cosh[(n-1)\beta(t-t_1)]} : \sigma = +1, \\ \pm\beta(t-t_1) : \sigma = 0, \\ \frac{1}{n-1} \ln \frac{\beta}{\sinh[(n-1)\beta|t-t_1|]} : \sigma = -1, \end{cases} \quad (34)$$

β, t_1, t_2 and c 's are integration constants which satisfy

$$c_0 = \frac{q+1}{\Delta} \tilde{c} - \frac{\sum_{\alpha=1}^p c_\alpha}{n-1}, \quad c'_0 = -\frac{\sum_{\alpha=1}^p c'_\alpha}{n-1},$$

$$\tilde{c}_\alpha = c_\alpha - \frac{\gamma^{(\alpha)} - q - 1}{\Delta} \tilde{c}, \quad \tilde{c}_\phi = c_\phi + \frac{(D-2)\epsilon a}{\Delta} \tilde{c}. \quad (35)$$

These must further obey the condition

$$\frac{1}{n-1} \left(\sum_{\alpha=1}^p c_\alpha \right)^2 + \sum_{\alpha=1}^p c_\alpha^2 + \frac{1}{2} c_\phi^2 = n(n-1)\beta^2. \quad (36)$$

The free parameters in our solutions are $c_\alpha, c'_\alpha (\alpha = 1, \dots, p), c_\phi, c'_\phi, t_1$ and t_2 . The time derivative of E gives the field strengths of antisymmetric tensor and in our convention they are given as

$$\left. \begin{aligned} e^{a\phi} *F \\ F \end{aligned} \right\} = \tilde{c} \sqrt{\frac{2(D-2)}{\Delta}} e^{-\sum_{\alpha \in q} c'_\alpha + \epsilon a c'_\phi / 2} dx^{\alpha_{q+2}} \wedge \dots \wedge dx^{\alpha_p} \wedge \text{Vol}(\Sigma_{n,\sigma}), \quad (37)$$

for electric (first line) and magnetic (second line) fields, where $\text{Vol}(\Sigma_{n,\sigma})$ is the unit volume form of the hypersurface $\Sigma_{n,\sigma}$ and $*$ represents dual. We can check that $\sqrt{\frac{2(D-2)}{\Delta}} = 1$ for SM- and SD-branes.

For the general S2-brane obtained from the solution (31) by putting $p = q+1 = 3, c \equiv c_1 = c_2 = c_3, c' \equiv c'_1 = c'_2 = c'_3$, we find that it takes the form (22) and (23) with

$$\delta(t) = [\cosh \tilde{c}(t-t_2)]^{3/\Delta} e^{g(t)+c_0 t+c'_0},$$

$$S(t) = [\cosh \tilde{c}(t-t_2)]^{(n+2)/2\Delta} e^{ng(t)/2+(n+2)(c_0 t+c'_0)/6}, \quad (38)$$

where

$$\tilde{c} = 3c - \frac{1}{2}c_\phi \epsilon a, \quad c_0 = \frac{3}{\Delta} \tilde{c} - \frac{3}{n-1}c, \quad c'_0 = -\frac{3}{n-1}c',$$

$$\beta = \sqrt{\frac{3(n+2)}{n(n-1)^2} c^2 + \frac{1}{2n(n-1)} c_\phi^2}. \quad (39)$$

4.2. Vacuum solution

The relation between \tilde{c} and c_α and c_ϕ in Eq. (32) is derived²⁵ under the assumption that we have the independent field strengths F . In the absence of these, we can disregard this relation and set \tilde{c} to zero. It is then easy to see that the solution (31) reproduces (22)-(24) for $p = q + 1 = 3, \sigma = -1, c = 1, c' = 0$ without dilaton ($c_\phi = 0$). The scale factor is simply (38) with $\tilde{c} = 0$ which coincides with (24).

The condition of the expansion (26) for the vacuum solution (24) is¹²

$$n_1(t) \equiv -1 - \sqrt{\frac{3n}{n+2}} \coth\left(\sqrt{\frac{3(n+2)}{n}} c(t-t_1)\right) > 0, \quad (40)$$

where we have also included parameters c, c' and t_1 . The condition (27) gives

$$\frac{3(n-1)}{(n+2) \sinh^2[\sqrt{3(n+2)/n} c(t-t_1)]} - n_1^2(t) > 0. \quad (41)$$

The parameter t_1 and c can be absorbed into the shift and rescaling of the time t . Hence without loss of generality, we can set $t_1 = 0$ and $c = 1$ (changing c gives the change in the scale of time). There is a singularity in $S(t)$ at $t = 0$, but the time η runs from 0 to infinity while t runs from $-\infty$ to 0, which is an infinite future for any event with $t < 0$ and hence the evolution of our universe can be restricted to $t < 0$.

We find for $n = 7$ that there is a certain period of negative time that the conditions (40) and (41) are satisfied.¹² The period of the accelerated expansion can be adjusted by changing the constant c , but this does not affect the resulting expansion factor. The scale factor vanishes in the infinite past, but diverges in the infinite future.

The expansion factor A during the accelerated expansion is given by the ratio of $S(t)$ at the starting time T_1 and ending time T_2 of the acceleration. The expansion factor, which does not change if the constant c is changed, is found to be

$$A = \frac{S(T_2)}{S(T_1)} \simeq 2.91. \quad (42)$$

This value is too small to explain the cosmological problems. Note that there is no parameter to improve A here.

The behavior of the size of the internal space is also examined. When the acceleration starts, $\delta(t)$ shrinks but eventually starts expanding, and the ratio of the sizes during the accelerated expansion is 2.18. As observed for SM2-brane case,²⁹ there is no stable point in the size of the internal space, and in the infinite future and past its size goes to infinity. This seems to be the common problem in the hyperbolic compactification.³⁰ We shall find this behavior in other cases.

Other internal spaces are also examined, but without any adjustable parameter it is found that neither flat nor spherical spaces give accelerating cosmologies as long as the vacuum Einstein equation is considered.¹³ Product spaces for the internal spaces are also examined, and suggestion for large e-foldings is given.³¹ However, there has not been given any concrete model with large e-folding. Generalization to other dimensions is also considered in Ref. 32.

4.3. SM2-brane

The SM2-brane in M-theory can be obtained from (31) by putting $p = q + 1 = 3, \epsilon = +1, a = 0, c_\phi = 0$ without dilaton and $\Delta = 3(n - 1)$. We also put $c \equiv c_1 = c_2 = c_3, c' \equiv c'_1 = c'_2 = c'_3$ and then $\beta = \frac{1}{n-1} \sqrt{\frac{3(n+2)}{n}}$ c is determined from (36). The solution (31) then gives

$$ds_d^2 = [\cosh 3c(t - t_2)]^{2/(n-1)} \left[-e^{2ng(t)-6c'/(n-1)} dt^2 + e^{2g(t)-6c'/(n-1)} d\Sigma_{n,\sigma}^2 + [\cosh 3c(t - t_2)]^{-2(n+2)/3(n-1)} e^{2c'} d\mathbf{x}^2 \right]. \quad (43)$$

This is the universe (22) and (23) with

$$\begin{aligned} \delta(t) &= [\cosh 3c(t - t_2)]^{1/(n-1)} e^{g(t)-3c'/(n-1)}, \\ S(t) &= [\cosh 3c(t - t_2)]^{(n+2)/6(n-1)} e^{ng(t)/2-(n+2)c'/(n-1)}. \end{aligned} \quad (44)$$

We now discuss three internal spaces (34) separately.

4.3.1. Hyperbolic internal space

The conditions (26) and (27) for $n = 7$ of our interest and hyperbolic internal space $\sigma = -1$ are (again shifting the time to set $t_1 = 0$)

$$n_2(t) \equiv \frac{3}{4} \tanh[3c(t - t_2)] - \frac{\sqrt{21}}{4} \coth(3\sqrt{3/7}ct) > 0, \quad (45)$$

$$\frac{9}{8} \left(\frac{1}{\cosh^2[3c(t - t_2)]} + \frac{1}{\sinh^2(3\sqrt{3/7}ct)} \right) - n_2^2(t) > 0. \quad (46)$$

Here we can again consider that our universe evolves only for $t < 0$. The qualitative behaviors of the left hand side of these Eqs. for $c = 1$ and $t_2 = 0$ are similar to those in the previous S-brane case, and we again find that there is a certain period of negative time that these conditions are satisfied.¹³

The expansion factor is found to be^{29,13}

$$A \simeq 2.17, \quad (47)$$

which is too small to explain the cosmological problems. However, here is a parameter t_2 in contrast to the vacuum solution, whose effect is examined. It is found that the typical behavior for positive t_2 is basically the same as $t_2 = 0$ case, but the period of the accelerated expansion changes slightly. For example, for $t_2 = 1$, we find the value of the expansion factor during the accelerated expansion improves:

$$A \simeq 3.13. \quad (48)$$

This is still not enough improvement for cosmological applications. Increasing the value of t_2 does not affect the numerical value of A much. The behavior for negative

t_2 is also basically the same, but the expansion factor is worse; one typically gets $A \sim 1.4$.

The behavior of the size of the internal space for $t_2 = 0$ is also similar. As observed in Ref. 29 there is no stable point in the size of the internal space, and in the infinite future its size goes to infinity. There is no significant change in this behavior if we change the parameter t_2 .

4.3.2. Flat internal space

The conditions (26) and (27) for $n = 7$ and flat internal space $\sigma = 0$ are

$$n_3(t) \equiv \frac{3}{4} \tanh[3c(t - t_2)] + \frac{\sqrt{21}}{4} > 0, \quad (49)$$

$$\frac{9}{8} \frac{1}{\cosh^2[3c(t - t_2)]} - n_3^2(t) > 0, \quad (50)$$

where we have chosen the plus sign in Eq. (34) since minus sign cannot give expanding universe.

Here since $S(t)$ does not have any singularity and is positive, the time t runs from $-\infty$ to $+\infty$ while the time η runs from 0 to $+\infty$ monotonously. The left hand side of these Eqs. for $c = 1$ and $t_2 = 0$ are examined and we again find that there is a certain period of negative time that these conditions are satisfied.¹³

The expansion factor in this case is found to be

$$A \simeq 1.35, \quad (51)$$

which is again too small to explain the cosmological problems. The conditions (49) and (50) depend only on $t - t_2$, so changing t_2 simply shifts the evolution of the spacetime and does not give any difference.

The behavior of the size of the internal space is also similar. There is no stable point in the size of the internal space, and in the infinite future its size goes to infinity.

4.3.3. Spherical internal space

The conditions (26) and (27) for $n = 7$ and spherical internal space $\sigma = +1$ are (again shifting the time to set $t_1 = 0$)

$$n_4(t) \equiv \frac{3}{4} \tanh[3c(t - t_2)] - \frac{\sqrt{21}}{4} \tanh(3\sqrt{3/7} ct) > 0, \quad (52)$$

$$\frac{9}{8} \left(\frac{1}{\cosh^2[3c(t - t_2)]} - \frac{1}{\cosh^2(3\sqrt{3/7} ct)} \right) - n_4^2(t) > 0. \quad (53)$$

The time ranges of η and t are the same as the flat internal space. It was first thought that there is no period of accelerated expansion. However when the parameter t_2 is changed, the results change. For $c = 1$ and $t_2 = -1$, we find that there is a certain period of negative time that the conditions (52) and (53) are satisfied, though the

universe begin contraction after some positive time. The scale factor $S(t)$ in (44) contracts on both ends $t \rightarrow \pm\infty$. The expansion factor is

$$A \simeq 1.35, \quad (54)$$

which is again too small to explain the cosmological problems. We have also checked that the typical behavior for negative t_2 is again basically the same as $t_2 = -1$ case, but the period of the accelerated expansion and the value of the expansion factor during the accelerated expansion change.

A different behavior is observed for positive t_2 . We find the acceleration occurs while the universe is already contracting for $t > 0$. There is no stable point in the size of the internal space, and in the infinite future its size goes to infinity. There is no significant change in this behavior if we further change the parameter t_2 .

The value of the expansion factor during the accelerated expansion in all these solutions are so small that they cannot give inflationary models at the early epoch. How about the present accelerating cosmology? The size of the internal space of these models is examined in Ref. 33 and it was concluded that it is too large to give a four-dimensional model of the present accelerating universe. Thus we would need some mechanism to keep the size of the internal space small, or consider a kind of warp factors to render the large extra dimensions harmless.³⁴ It is desirable to find modification of these solutions without invoking some special objects, which need further explanation why it is natural to consider such objects. Such approach would be similar to introducing scalar field with suitable potential, and in a sense contradicts our spirit to use our fundamental theory. We hope to derive an accelerating cosmologies in a well-defined setting without the need of further explanation, and this problem of stabilization of the size of the internal space is a subject left for future study.

4.4. SD2-brane

In this section, we discuss accelerating cosmologies from SD2-brane.^{13,35}

We first show how the SD2-brane can be obtained by dimensional reduction from SM2-branes.¹³ Though it is possible to examine general dimensions, we restrict ourselves to $D = 11$ here.

The ten-dimensional metric in the string frame is obtained from the eleven-dimensional one by the relation³⁶

$$ds_{11}^2 = e^{-2\phi/3} ds_{10,s}^2 + e^{4\phi/3} dx_{10}^2. \quad (55)$$

In our solutions (31), we consider SM2-brane by taking $q = 2$, but we set $p = 4$ with one extra coordinate outside SM2-brane, in which direction we make dimensional reduction. We choose x_1, x_2, x_3 to be the SD2-brane world-volume and x_4 the direction of dimensional reduction. We further set $c \equiv c_1 = c_2 = c_3, c' \equiv c'_1 = c'_2 = c'_3$ but leave c_4 and c'_4 arbitrary. There is no dilaton which comes from the metric

as (55) upon dimensional reduction, and we put $a = c_\phi = 0$. This gives

$$\begin{aligned} \phi &= \frac{1}{4} \ln[\cosh 3c(t - t_2)] + \frac{3}{4}(c + 2c_4)t + \frac{3}{2}c'_4, \\ ds_{10,s}^2 &= e^{2\phi/3} [\cosh 3c(t - t_2)]^{1/3} \left[e^{-(c+2c_4)t/5 - 2(3c' + c'_4)/5} \left\{ -e^{12g(t)} dt^2 + e^{2g(t)} d\Sigma_{6,\sigma}^2 \right\} \right. \\ &\quad \left. + [\cosh 3c(t - t_2)]^{-1} e^{2c'} d\mathbf{x}^2 \right]. \end{aligned} \quad (56)$$

In the Einstein frame $ds_{10,E}^2 = e^{-\phi/2} ds_{10,s}^2$, this reduces to

$$\begin{aligned} ds_{10,E}^2 &= [\cosh 3c(t - t_2)]^{3/8} \left[e^{-3(c+2c_4)t/40 - 3(8c' + c'_4)/20} \left\{ -e^{12g(t)} dt^2 + e^{2g(t)} d\Sigma_{6,\sigma}^2 \right\} \right. \\ &\quad \left. + [\cosh 3c(t - t_2)]^{-1} e^{(c+2c_4)t/8 + (8c' + c'_4)/4} d\mathbf{x}^2 \right]. \end{aligned} \quad (57)$$

The scale factor $S(t)$ and $\delta(t)$ are given as

$$\begin{aligned} \delta(t) &= [\cosh(16\alpha/5)(t - t_2)]^{3/16} e^{g(t) - 3c_1 t/64 - 3\tilde{c}'/4}, \\ S(t) &= [\cosh(16\alpha/5)(t - t_2)]^{1/4} e^{3g(t) - c_1 t/16 - \tilde{c}'}, \\ \phi &= \frac{1}{4} \ln[\cosh(16\alpha/5)(t - t_2)] + \frac{15}{16}c_1 t + c'_\phi, \end{aligned} \quad (58)$$

where we have also put

$$\tilde{c}' = \frac{8c' + c'_4}{10}, \quad c'_\phi = \frac{3}{2}c'_4. \quad (59)$$

On the other hand, if we consider the SD2-brane with $D = 10, q = 2, p = 3, n = 6, a = \frac{1}{2}, \epsilon = +1$ in Eq. (31), we get precisely (58) with different parametrization. Continuing this reduction, one should leave the constants c_α arbitrary in order to get general lower-dimensional solutions.

The scale factors (58) almost coincide with those for SM2-brane in (44) if we choose $n = 6$, though the power of the \cosh is slightly different and there is an additional $c_1 t$ term in the exponent. However, the difference is small and one may expect that qualitative features remain the same. In fact this is explicitly confirmed in Ref. 13.

One potential subtlety in this picture of SD2-brane is that the string coupling is given by the dilaton as e^ϕ , and it may diverge for the solution (58) for certain range of parameters. If this happens, quantum effects dominate and we cannot simply trust the result. This can be avoided for hyperbolic internal space but it becomes more serious problem for other internal space. For instance, for flat internal space it is found that the dilaton (58) always gives strong string coupling somewhere for large $|t|$ whatever the choice of the parameters are, and it is not clear if this classical analysis is valid. The strong coupling limit is the one in which eleventh dimension becomes large.³⁷ This case is better described in the SM2-brane picture, with large eleventh dimension. This is what we have already analyzed in the preceding section.

4.5. Intuitive approach using effective four-dimensional potential

So far we have seen that the accelerating behavior is possible for time-dependent internal space, and this is possible only for the hyperbolic internal space for vacuum solution, but it is found that S-brane solutions give this behavior for flat and compact spaces. Why is this so? In fact, there is a very intuitive way to understand this, using the effective four-dimensional potential.³⁸

Let us consider the product space of d -dimensional universe and $\mathbb{M}_1 \times \mathbb{M}_2 \times \mathbb{M}_3$ with dimensions m_1, m_2, m_3 and signatures of curvature $\sigma_1, \sigma_2, \sigma_3$. Products of one or two spaces for extra dimensions can be obtained by setting m 's to zero. The ansatz of the metric for the full spacetime is

$$ds^2 = e^{2\phi(x)} ds_d^2 + \sum_{i=1}^3 e^{2\phi_i(x)} d\Sigma_{m_i, \sigma_i}^2, \quad (60)$$

where we have chosen the Einstein frame by setting

$$\phi = - \sum_i m_i \phi_i / (d-2). \quad (61)$$

The kinetic terms for the scalars corresponding to the radii of each internal space in the effective theory are given by

$$K = \frac{\rho + p}{2} = \sum_{i=1}^3 \frac{m_i(m_i + d - 2)}{2(d-2)a^2} \dot{\phi}_i^2 + \sum_{i>j=1}^3 \frac{m_i m_j}{(d-2)a^2} \dot{\phi}_i \dot{\phi}_j - \sigma_0 \frac{d-2}{2a^2}. \quad (62)$$

They can be diagonalized and normalized by a field redefinition

$$\begin{aligned} \psi_1 &= \sqrt{\frac{m_1(m_1 + d - 2)}{d-2}} \left[\phi_1 + \frac{1}{m_1 + d - 2} (m_2 \phi_2 + m_3 \phi_3) \right], \\ \psi_2 &= \sqrt{\frac{m_2(m_1 + m_2 + d - 2)}{m_1 + d - 2}} \left[\phi_2 + \frac{m_3}{m_1 + m_2 + d - 2} \phi_3 \right], \\ \psi_3 &= \sqrt{\frac{m_3(m_1 + m_2 + m_3 + d - 2)}{m_1 + m_2 + d - 2}} \phi_3, \end{aligned} \quad (63)$$

with the result

$$K = \frac{1}{2} \sum_{i=1}^3 \dot{\psi}_i^2 - \epsilon_0 \frac{d-2}{2a^2}, \quad (64)$$

$$V = - \sum_{i=1}^3 \sigma_i \frac{m_i(m_i - 1)}{2} e^{\sum_a 2M_{ia} \psi_a} - \sigma_0 \frac{(d-2)^2}{2a^2}, \quad (65)$$

where the matrix M_{ia} is given by

$$M_{ia} = \begin{pmatrix} -\sqrt{\frac{m_1+d-2}{(d-2)m_1}} & 0 & 0 \\ -\sqrt{\frac{m_1}{(d-2)(m_1+d-2)}} & -\sqrt{\frac{m_1+m_2+d-2}{m_2(m_1+d-2)}} & 0 \\ -\sqrt{\frac{m_1}{(d-2)(m_1+d-2)}} & -\sqrt{\frac{m_2}{(m_1+d-2)(m_1+m_2+d-2)}} & -\sqrt{\frac{m_1+m_2+m_3+d-2}{m_3(m_1+m_2+d-2)}} \end{pmatrix}. \quad (66)$$

To study the properties of the potential, it is more convenient to define new independent fields as

$$\begin{aligned}\varphi_1 &\equiv 2\sqrt{\frac{m_1+d-2}{m_1(d-2)}}\psi_1, \\ \varphi_2 &\equiv 2\sqrt{\frac{m_1}{(d-2)(m_1+d-2)}}\psi_1 + 2\sqrt{\frac{m_1+m_2+d-2}{m_2(m_1+d-2)}}\psi_2, \\ \varphi_3 &\equiv 2\sqrt{\frac{m_1}{(d-2)(m_1+d-2)}}\psi_1 + 2\sqrt{\frac{m_2}{(m_1+d-2)(m_1+m_2+d-2)}}\psi_2 \\ &\quad + 2\sqrt{\frac{m_1+m_2+m_3+d-2}{m_3(m_1+m_2+d-2)}}\psi_3.\end{aligned}\tag{67}$$

and the effective potential (65) becomes¹⁴

$$V = -\sum_{i=1}^3 \sigma_i \frac{m_i(m_i-1)}{2} e^{-\varphi_i} - \sigma_0 \frac{(d-2)^2}{2a^2}.\tag{68}$$

Clearly the potential is unbounded if any one of the σ_i 's is positive in that direction and positive with minimum at infinity if the σ_i is negative. The potentials for positive and negative σ_i are shown by solid lines in Fig. 1. However, if we add contributions from antisymmetric tensors, this is modified. For instance, the contribution of the 4-form field in eleven-dimensional supergravity is

$$\Delta V = b^2 \exp \left[-\frac{(d-1)(m_1\varphi_1 + m_2\varphi_2 + m_3\varphi_3)}{m_1 + m_2 + m_3 + d-2} \right],\tag{69}$$

where b is a constant. Now the potential is bounded from below if $\frac{(d-1)m_i}{m_1+m_2+m_3+d-2} > 1$ for the direction $\sigma_i = +1$, as shown by the dashed line in Fig. 1. (There is no requirement and no minimum in the direction $\sigma_i = -1, 0$.) For $d = 4$, this is easily satisfied. In this case, there will be local minimum in the direction with $\sigma_i = +1$, but the potential minimum is always negative.

Single internal space is discussed in Ref. 38, with only one scalar field φ_1 , $d = 4$ and $m_1 = n, m_2 = m_3 = 0$. Suppose that the field starts at a large value of $\varphi_1 > 0$,

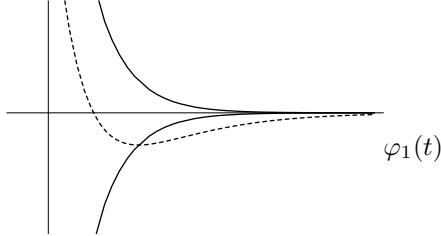


Fig. 1. Potential for scalar field for $\sigma = \pm 1$ with $b = 0$ (solid) and $b \neq 0, \sigma = +1$ (dashed).

with large negative velocity $\dot{\varphi}_1$. This is a phase of decelerated expansion. If the

kinetic energy of φ_1 is large enough, the field will run up the exponential potential hill at later time, and it will stop. Around this, the potential dominates and the universe is in the accelerating phase. Soon after this, the field starts rolling back down the hill and the universe gets back to the decelerating phase.

It is clear that this behavior is obtained for the vacuum without flux ($b = 0$) only if $\epsilon = -1$, namely for hyperbolic space, whereas it can be obtained for the S-brane solutions with flux for all kinds of internal spaces. This is the reason why this solution is obtained only for the hyperbolic internal space in Townsend-Wohlfarth solution. However, it is difficult to get big expansion even in the present case since the exponential potential is too steep to keep the field at the top for long time. This remains true if we take the internal space to be the product space.¹⁴ In order to have big expansion, we should have local minimum at positive value, where our universe stays for a while and expands, and then decays to lower value. However the no-go theorem claims that there is no such minimum.

It is also interesting that a potential with local minimum (though at negative value) is obtained in the Einstein gravity coupled to gauge fields, providing a mechanism for stabilizing the size of extra dimensions. How to stabilize the size of extra dimensions is an issue no less important than obtaining inflation. This is a direction worthy of further exploration.

Another interesting research direction is to try to obtain inflation in the present picture by introducing matter fields into the solutions. This could be done, for example, by considering D-branes in the solutions. Such solutions have recently been discussed in Ref. 39. Related models using D-branes are proposed⁴⁰ and have been extensively studied. Brane and anti-brane systems are also a focus of extensive study.⁴¹ Such systems naturally contain tachyons which may drive accelerating behavior.⁴² Unfortunately it is pointed out that it is rather difficult to obtain inflation and it can occur only at super-Planckian densities where the effective four-dimensional field theory is not applicable.⁴³

A scalar field with a positive potential which yields an accelerating universe has been named “quintessence”.^{44,45} Supergravity models which exhibit such properties are discussed in Ref. 46, and rather systematic study of theories with a scalar with exponential potential and connection with gauged supergravities are presented in Ref. 47. Possible problems with this approach in the attempt at making sense of quantum theory of string theories are pointed out in Ref. 48. More exotic models with scalar fields with higher order kinetic terms (called “ k -essence”) are shown to lead to power-law expansion,⁴⁹ but the problem with this approach is how to justify the existence of such a scalar field.

5. Eternal Expansion

Considering the difficulty in obtaining large e-foldings in the simple S-brane solutions, it is desirable to investigate whatever modifications to improve it. Here we present one interesting solution which exhibits eternal expansion.¹⁴

5.1. An exact critical solution

Let the higher-dimensional geometry be $\mathbb{R} \times \mathbb{H}_d \times \mathbb{H}_m$. The metric ansatz is

$$ds^2 = e^{-\frac{2m}{d-2}\phi(t)} (-dt^2 + a(t)^2 ds_{Hd}^2) + e^{2\phi(t)} ds_{Hm}^2. \quad (70)$$

The size of \mathbb{H}_m corresponds to a scalar field in the effective theory on $\mathbb{R} \times \mathbb{H}_d$. Here we write the (non-trivial) components of the Einstein tensor for arbitrary σ_0, σ_1 :

$$G_{00} = - \left[\frac{\lambda}{2} \dot{\phi}^2 - \sigma_1 \frac{m(m-1)}{2} e^{-\frac{2\lambda}{m}\phi(t)} - \frac{(d-1)(d-2)}{2} \left(H^2 + \frac{\sigma_0}{a^2} \right) \right], \quad (71)$$

$$G_{xx} = -a^2 \left[\frac{\lambda}{2} \dot{\phi}^2 + \sigma_1 \frac{m(m-1)}{2} e^{-\frac{2\lambda}{m}\phi(t)} + \frac{(d-2)(d-3)}{2} \left(H^2 + \frac{\sigma_0}{a^2} \right) + (d-2) \frac{\ddot{a}}{a} \right],$$

$$\begin{aligned} G_{ii} = & \left[\frac{\lambda}{m} \left(\ddot{\phi} + (d-1) H \dot{\phi} \right) - \frac{\lambda}{2} \dot{\phi}^2 - \sigma_1 \frac{(m-1)(m-2)}{2} e^{-\frac{2\lambda}{m}\phi(t)} \right. \\ & \left. - \frac{(d-1)(d-2)}{2} \left(H^2 + \frac{\sigma_0}{a^2} \right) - (d-1) \frac{\ddot{a}}{a} \right] e^{\frac{2\lambda}{m}\phi(t)}, \end{aligned} \quad (72)$$

where $\lambda \equiv \frac{m(m+d-2)}{(d-2)}$. The metric on $\mathbb{R} \times \mathbb{H}_{d-1} \times \mathbb{H}_m$ space takes the values $\sigma_0 = \sigma_1 = -1$. Putting $d = 4$ and by a change of variable

$$\phi = \sqrt{\frac{2}{m(m+2)}} \psi + \frac{1}{m+2} \ln(m(m-1)), \quad (73)$$

we write the Friedman equation and the wave equation for ψ as

$$3H^2 = \frac{1}{2} \dot{\psi}^2 + \frac{1}{2} e^{-c\psi} + \frac{3}{a^2}, \quad c = \sqrt{\frac{2(m+2)}{m}}, \quad (74)$$

$$\ddot{\psi} + 3H\dot{\psi} - \frac{c}{2} e^{-c\psi} = 0. \quad (75)$$

It is straightforward to obtain the critical (in the sense that the power exponent of the scale factor is on the boundary to be inflationary) solution with $\ddot{a} = 0$:

$$a = \sqrt{\frac{m+2}{2}} t, \quad \psi = \frac{2}{c} \ln(t) + \frac{1}{c} \ln \left(\frac{c^2}{8} \right). \quad (76)$$

5.2. Scalar perturbations

A tiny perturbation to the exact solution is found to lead to interesting behavior of an accelerating phase. Consider a small perturbation around the solution (76). Let

$$a = a_0 + a_1, \quad \psi = \psi_0 + \psi_1, \quad (77)$$

where a_0 and ψ_0 are given by (76). It follows that the Hubble parameter is

$$H = H_0 + H_1, \quad H_0 = \frac{1}{t}, \quad H_1 = \frac{\dot{a}_1}{a_0} - H_0 \frac{a_1}{a_0}, \quad (78)$$

to the first order approximation. Expanding Eqs. (74) and (75) and keeping first order terms only, we get

$$6H_0H_1 = \dot{\psi}_0\dot{\psi}_1 - \frac{c}{2}e^{-c\psi_0}\psi_1 - 6\frac{a_1}{a_0^3}, \quad (79)$$

$$\ddot{\psi}_1 + 3H_0\dot{\psi}_1 + 3H_1\dot{\psi}_0 + \frac{c^2}{2}e^{-c\psi_0}\psi_1 = 0, \quad (80)$$

along with

$$2\dot{\psi}_0\dot{\psi}_1 + \frac{3\ddot{a}_1}{a_0} + \frac{c}{2}e^{-c\psi_0}\psi_1 = 0. \quad (81)$$

These linear equations are solved with the following solutions

$$a_1 = At^n, \quad \psi_1 = \gamma At^{n-1}, \quad (82)$$

where

$$\gamma = \frac{3(1-n)}{2\sqrt{m}}; \quad n = \pm\sqrt{\frac{m-6}{m+2}}. \quad (83)$$

These give real solutions only if

$$m > 6. \quad (84)$$

Note that $m = 6$ or $n = 0$ is excluded because it is just a zero mode corresponding to time-shift symmetry. There are solutions with eternally accelerating expansion when $m \geq 7$, although $m = 7$ is the most interesting case since $m = 7$ (together with our spacetime 4 dimensions) is precisely the dimension in which M-theory lives. This coincidence suggests that the approach is worth serious consideration.

For the case $m = 7$, we have

$$n = \frac{1}{3}, -\frac{1}{3}; \quad \gamma = \frac{1}{\sqrt{7}}, \frac{2}{\sqrt{7}}. \quad (85)$$

Examining the $n = 1/3$ solution we find that

$$a_1 = At^{1/3}, \quad \ddot{a} = -\frac{2A}{9t^{5/3}}, \quad (86)$$

so this gives positive acceleration for $A < 0$. However, as time increases, a_1 grows and perturbative expansion is no longer valid. We cannot claim eternally accelerating expansion for this solution without further analysis. The other solution $n = -1/3$,

$$a_1 = At^{-1/3}, \quad \ddot{a} = \frac{4A}{9t^{7/3}}, \quad (87)$$

gives a positive acceleration for $A > 0$. As time increases, a_1 approaches to zero and our perturbative calculation remains valid. We find eternally accelerating expansion for this case. This behavior is also confirmed by numerical evaluation without recourse to perturbation.

For product space compactifications, it is generally difficult to find exact solutions for the coupled Einstein equations unless the internal space is a product of

flat spaces and at most one nontrivial curved space or they all are of the same type. It would be interesting to find the exact solution corresponding to the solution we have obtained here with eternally accelerating expansion and see if the inflation is further increased at early times and not just at late times.

6. Cosmologies with exponential potentials

In Sec. 4.4, we have seen that the cosmological solutions can be analyzed by using the effective theories in four dimensions, which typically contains exponential potentials of scalar fields. This may be regarded as an alternative approach using the effective four-dimensional theories. In this section, we briefly discuss this subject.

The cosmological solutions of the system with a single scalar field can be systematically studied and classified.^{50,51,52} The field equations to be studied are Eqs. (74) and (75). We define a new time parameter

$$d\tau = e^{-\frac{c}{2}\psi} dt, \quad (88)$$

and set $a(t) = e^{\alpha(\tau)}$. Denoting the derivative with respect to τ by a dot, Eqs (74) and (75) for hyperbolic internal space reduce to

$$3\dot{\alpha}^2 - \frac{1}{2}\dot{\psi}^2 = \frac{1}{2} - \sigma_0 \frac{3}{a^2} e^{c\psi}, \quad c = \sqrt{\frac{2(m+2)}{m}}, \quad (89)$$

$$\ddot{\psi} - \frac{c}{2}\dot{\psi}^2 + 3\dot{\alpha}\dot{\psi} = \frac{c}{2}. \quad (90)$$

Note that the constant is restricted to the range

$$\sqrt{\frac{18}{7}} \geq c > \sqrt{2}, \quad (91)$$

for $m \geq 7$. When the 4-form flux is included in M-theory, we find from Eq. (69) that there appears a potential of the form

$$b^2 e^{-\frac{6}{c}\psi}. \quad (92)$$

The range of the coefficient is

$$\frac{6}{\sqrt{2}} > \frac{6}{c} \geq \sqrt{14}, \quad (93)$$

for $m \geq 7$.

For $\sigma_0 = 0$, Eq. (89) defines a hyperbola which separates the $\sigma_0 = -1$ and $\sigma_0 = +1$ trajectories. We are interested in the $\dot{\alpha} > 0$ branch where we have expanding universe. Let us parametrize this branch as

$$\dot{\alpha} = \frac{1}{\sqrt{24}}(\xi + \xi^{-1}), \quad \dot{\psi} = \frac{1}{2}(\xi - \xi^{-1}), \quad (\xi > 0). \quad (94)$$

Equation (90) then becomes

$$\dot{\xi} = \frac{1}{4}[c + \sqrt{6} + (c - \sqrt{6})\xi^2]. \quad (95)$$

We are now going to study several cases separately.

6.1. $c < \sqrt{6}$

In this case (which is within the range (91)), there is a fixed point solution

$$\xi = \xi_0 \equiv \left(\frac{\sqrt{6} + c}{\sqrt{6} - c} \right)^{\frac{1}{2}}. \quad (96)$$

Since ξ is a constant, Eqs. (94) shows that α and ψ are linear in τ :

$$\alpha = \frac{\tau}{\sqrt{6 - c^2}} + \text{const.}, \quad \psi = \frac{c}{\sqrt{6 - c^2}}\tau + \text{const..} \quad (97)$$

We deduce from Eq. (88) that

$$t \sim e^{\frac{c^2}{2\sqrt{6-c^2}}\tau}, \quad (98)$$

and hence

$$e^\psi \sim t^{2/c}, \quad a \sim t^{2/c^2}. \quad (99)$$

This is an expanding solution by power law.

There are two solutions to Eq. (95):

$$(i) \xi = \xi_0 \tanh \gamma\tau, \quad (ii) \xi = \xi_0 \coth \gamma\tau, \quad \gamma \equiv \frac{\sqrt{6 - c^2}}{4}. \quad (100)$$

Only the solution (i) includes $\dot{\xi} = 0$, hence $\dot{\psi} = 0$. This solution undergoes a period of accelerating expansion. We find

$$a^{\sqrt{6}} \propto (\cosh \gamma\tau)^{\lambda_-} (\sinh \gamma\tau)^{\lambda_+}, \quad e^\psi \propto (\cosh \gamma\tau)^{\lambda_-} (\sinh \gamma\tau)^{-\lambda_+}, \quad (101)$$

where

$$\lambda_{\pm} = \frac{2}{\sqrt{6} \pm c}. \quad (102)$$

This solution exhibits a decelerating epoch and then passes through a period of acceleration and finally approaches the attractor (99). In case (ii), one has

$$a^{\sqrt{6}} \propto (\cosh \gamma\tau)^{\lambda_+} (\sinh \gamma\tau)^{\lambda_-}, \quad e^\psi \propto (\cosh \gamma\tau)^{-\lambda_+} (\sinh \gamma\tau)^{\lambda_-}. \quad (103)$$

If $c = \sqrt{2}$ is allowed, the power-law solution (99) gives $a \sim t$ with zero acceleration. In this case, the above solution approaching this asymptotically must be eternally accelerating solution, which is similar to that found in Sec. 5.

6.2. $c > \sqrt{6}$

The case for $c > \sqrt{6}$ is similar. The solution to Eq. (95) is

$$\xi = \left(\frac{c + \sqrt{6}}{c - \sqrt{6}} \right)^{\frac{1}{2}} \tan \omega\tau, \quad \omega \equiv \frac{\sqrt{c^2 - 6}}{4}, \quad (104)$$

where $0 < \tau < \pi/2$. Eqs. (94) can be integrated to give

$$a^{\sqrt{6}} \propto (\cos \omega\tau)^{\lambda_-} (\sin \omega\tau)^{\lambda_+}, \quad e^\psi \propto (\cos \omega\tau)^{\lambda_-} (\sin \omega\tau)^{-\lambda_+}. \quad (105)$$

The asymptotic behavior for $\log t \rightarrow \pm\infty$ is

$$a \sim t^{1/3}, \quad e^\psi \sim t^{\pm 2/\sqrt{6}}. \quad (106)$$

There is also a period of acceleration.

6.3. $c = \sqrt{6}$

We have simply $\xi = \frac{\sqrt{6}}{2}\tau$ and

$$a^3 \sim \tau^{1/2} e^{\frac{3}{8}\tau^2}, \quad e^{\frac{\sqrt{6}}{2}\psi} \sim \tau^{-1/2} e^{\frac{3}{8}\tau^2}. \quad (107)$$

The asymptotic behavior is power-law with logarithmic corrections.

Solutions with flux in this approach have been discussed in Ref. 53 and general discussions are given in Ref. 54. A more systematic method to solve field equations by using new variable is discussed in Ref. 55. Power-law solutions of inflationary nature with exponential potential is discussed in Ref. 56.

6.4. Case of multi-scalars

The cosmological solutions with multi-scalars and hence with multi-exponential potentials are more involved. Some interesting solutions for this case are obtained in Refs. 57, 58, 59, 60. Solutions to exponential potentials with pressureless baryonic matter are discussed in Ref. 61. General discussions of attractors and repeller solutions are given in Refs. 62, to which we refer the reader for more details. From the string theory point of view, it is natural to consider antisymmetric tensors together with scalar fields with exponential potential. Solutions for such systems are studied and discussed in Ref. 63. Similar system of antisymmetric tensors and scalars is investigated in Ref. 64 for generating large four dimensions.

7. Generalized de Sitter Solutions from Theories with Higher-order Terms

We note that the scale when the acceleration occurs in the time-dependent solutions related to S-branes is basically governed by the Planck scale in the higher (ten or eleven) dimensions. When dealing with phenomena at such high energy, it is expected that we cannot ignore higher order corrections to the lowest Einstein-Hilbert (EH) term in the theories at least in the early universe. In fact there are terms of higher orders in the curvature to the lowest effective supergravity action coming from superstrings or M-theory. In four dimensions, many studies have been done with such correction terms.⁴

The cosmological models in higher dimensions were also studied intensively in the 80's by many authors.^{65,66,67,68,69,70,71} Higher order terms in scalar curvature

were considered in Refs. 66, 71, 72. In particular, it was shown that these theories can be mapped to a gravity system coupled with a scalar field by a conformal transformation,^{66,71} and hence are very easy to deal with.

The higher-order curvature terms called Lovelock gravity⁷³ were also considered in higher-dimensional cosmology.^{74,75,76} The lowest higher order corrections are expected to appear in this form called Gauss-Bonnet (GB) terms which are the special combination without ghosts.⁷⁷ In fact, it is known that this combination appears in the low-energy effective theories of heterotic strings,^{78,79} and extensive study of these theories was carried out.^{67,68} It was shown⁶⁸ that there are two exponentially expanding solutions, which may be called generalized de Sitter solutions since the size of the internal space depends on time (otherwise there is no solution of this type). In both solutions, the external space inflates, while the internal space shrinks exponentially. (There are also two time-reversed solutions, i.e. the external space shrinks exponentially but the internal space inflates.) One solution is stable and the other is unstable. Since the present universe is not in the phase of de Sitter expansion with this energy scale, we cannot use the stable solution for a realistic universe. If we adopt the unstable solution, on the other hand, we may not find sufficient inflation unless we fine-tune the initial values.

However, these powers of scalar curvature or Lovelock gravity are not the types of corrections arising in type II superstrings or M-theory. In particular, it is known that the coefficient of the GB terms vanishes and the first higher order corrections start with R^4 terms (one is the fourth order Lovelock gravity and the other contains higher derivatives).⁸⁰ Probably due to the complicated nature of these corrections, this case was not studied and so it is important to examine how these corrections in the fundamental theories modify the above cosmological models and whether we can get interesting cosmological scenario with large e-foldings. The solutions to the vacuum Einstein equations with these higher order corrections are studied.¹⁵

Before proceeding to our discussions of these corrected theories, we note that the realization of de Sitter solutions in superstring theories is notoriously difficult. As the no-go theorem indicates, accelerating expansion, which is typical of de Sitter solutions, cannot be obtained at the stationary minima in superstring or supergravity theories. Various attempts to avoid this theorem are made by considering warped solutions for noncompact gauged supergravities,⁸¹ by including hypermultiplet in gauged supergravities,⁸² or by including nonperturbative effects in heterotic M-theory.⁸³ Note that most of these attempts require corrections to the lowest supergravity theories. We shall show that the quantum corrections in M-theory and type II superstrings mentioned above also give generalized de Sitter solutions without adding anything special. One thing worth mentioning is that this class of solutions do not exist if we keep the size of the internal space constant, reminiscent of the no-go theorem. Moreover this de Sitter phase decays after some period of inflation because of the small instability in the solutions. This is a very interesting mechanism to end the inflation naturally.

We consider the low-energy effective action for the superstrings and/or M-

theory:

$$S = S_{\text{EH}} + S_{\text{GB}} + S_4 + S_S, \quad (108)$$

where

$$S_{\text{EH}} = \frac{1}{2\kappa_D^2} \int d^D x \sqrt{-g} R, \quad (109)$$

$$S_{\text{GB}} = \frac{\alpha}{2\kappa_D^2} \int d^D x \sqrt{-g} G, \quad (110)$$

$$S_4 = \frac{\beta}{2\kappa_D^2} \int d^D x \sqrt{-g} \tilde{E}_8, \quad (111)$$

$$S_S = \frac{\gamma}{2\kappa_D^2} \int d^D x \sqrt{-g} \tilde{J}_0. \quad (112)$$

$$\tilde{E}_8 = -\frac{1}{2^4 \times 3!} \epsilon^{\alpha\beta\gamma\mu_1\nu_1\dots\mu_4\nu_4} \epsilon_{\alpha\beta\gamma\rho_1\sigma_1\dots\rho_4\sigma_4} R^{\rho_1\sigma_1}_{\mu_1\nu_1} \dots R^{\rho_4\sigma_4}_{\mu_4\nu_4}, \quad (113)$$

$$\tilde{J}_0 = R^{\lambda\mu\nu\kappa} R_{\alpha\mu\nu\beta} R_\lambda^{\rho\sigma\alpha} R^\beta_{\rho\sigma\kappa} + \frac{1}{2} R^{\lambda\kappa\mu\nu} R_{\alpha\beta\mu\nu} R_\lambda^{\rho\sigma\alpha} R^\beta_{\rho\sigma\kappa}. \quad (114)$$

Here we have dropped contributions from forms, κ_D^2 is a D -dimensional gravitational constant. For the heterotic string, the leading correction is given by the GB terms with the coefficient:^{78,79}

$$\alpha = \frac{1}{8}\alpha', \quad (115)$$

if we keep the dilaton constant, where α' is the Regge slope parameter. For the M-theory in 11 dimensions, the coefficient for the GB terms α vanishes, so we should consider forth order terms with the coefficients:⁸⁰

$$\beta = -\frac{\kappa_{11}^2 T_2}{3^2 \times 2^9 \times (2\pi)^4}, \quad \gamma = -\frac{\kappa_{11}^2 T_2}{3 \times 2^4 \times (2\pi)^4}, \quad (116)$$

where $T_2 = (2\pi^2/\kappa_{11}^2)^{1/3}$ is the membrane tension. Type II superstring has the same couplings in 10 dimensions, so we can discuss this case if we keep the dilaton field constant, but we consider 11D theory in this paper. Here we should note that contributions of the Ricci tensor $R_{\mu\nu}$ and scalar curvature R are not included in the fourth-order corrections (112) because these terms are not uniquely fixed.

Since we are interested in a cosmological time-dependent solution, we take the metric of our spacetime as

$$ds_{11}^2 = -e^{2u_0(t)} dt^2 + e^{2u_1(t)} \sum_{i=1}^3 (dx^i)^2 + e^{2u_2(t)} \sum_{a=5}^{11} (dy^a)^2, \quad (117)$$

where we assume that the external 3-space and the internal 7-space are flat. Taking variation of the action with respect to u_0 , u_1 , and u_2 , we obtain three basic equations, whose explicit forms are summarized in Appendix.

7.1. Generalized de Sitter Solutions

In cosmology, de Sitter inflationary expanding spacetime is the most interesting solution in the early universe. We study such solutions to the M-theory with higher order corrections with $\alpha = 0$ and other coefficients given in (116).¹⁵ We can also consider EH theory with corrections of GB term, but these solutions are already known.⁶⁸ Extensive study of the generalized de Sitter and power-law solutions is also made for the external and internal spaces with nontrivial curvatures and more inflationary solutions are found,¹⁶ but here we discuss only flat spaces.

Assuming the metric form of a generalized de Sitter spacetime as

$$u_0 = 0, \quad u_1 = \mu t, \quad u_2 = \nu t, \quad (118)$$

where μ and ν are some constants, we obtain three algebraic equations from the general field equations given in Appendix:

$$\begin{aligned} & \mu^2 + 7\mu\nu + 7\nu^2 + 20160\beta\mu\nu^5 [7\mu^2 + 7\mu\nu + \nu^2] \\ & - 7\gamma [12\mu^8 + 7\mu^2\nu^2(\mu^2 + \nu^2 + \mu\nu)^2 + 168\nu^8 + 7\mu^4\nu^2(2\mu + \nu)^2 + 21\mu^2\nu^4(\mu + 2\nu)^2] \\ & + 4\gamma(3\mu + 7\nu) [6\mu^7 + 42\nu^7 + 7\mu^2\nu^2(\mu + \nu)(\mu^2 + \nu^2 + \mu\nu)] = 0, \end{aligned} \quad (119)$$

$$\begin{aligned} & 3\mu^2 + 14\mu\nu + 28\nu^2 + 20160\beta\nu^5 [6\mu^3 + 24\mu^2\nu + 14\mu\nu^2 + \nu^3] \\ & + 3\gamma [12\mu^8 + 7\mu^2\nu^2(\mu^2 + \nu^2 + \mu\nu)^2 + 168\nu^8 + 7\mu^4\nu^2(2\mu + \nu)^2 + 21\mu^2\nu^4(\mu + 2\nu)^2] \\ & - 2\gamma\mu(3\mu + 7\nu) [48\mu^6 + 7\nu^2(3\mu^2 + 2\mu\nu + \nu^2)(\mu^2 + \nu^2 + \mu\nu) \\ & + 14\mu^2\nu^2(2\mu + \nu)(3\mu + \nu) + 42\nu^4(\mu + 2\nu)(\mu + \nu)] \end{aligned} \quad (120)$$

$$+ 2\gamma\mu(3\mu + 7\nu)^2 [12\mu^5 + 7\nu^2(\mu + \nu)(\mu^2 + \nu^2 + \mu\nu)] = 0, \quad (120)$$

$$\begin{aligned} & 2\mu^2 + 6\mu\nu + 7\nu^2 + 2880\beta\mu\nu^4 [15\mu^3 + 46\mu^2\nu + 38\mu\nu^2 + 6\nu^3] \\ & + \gamma [12\mu^8 + 7\mu^2\nu^2(\mu^2 + \nu^2 + \mu\nu)^2 + 168\nu^8 + 7\mu^4\nu^2(2\mu + \nu)^2 + 21\mu^2\nu^4(\mu + 2\nu)^2] \\ & - 2\gamma\nu(3\mu + 7\nu) [96\nu^6 + \mu^2(\mu^2 + 2\mu\nu + 3\nu^2)(\mu^2 + \nu^2 + \mu\nu) \\ & + 2\mu^4(2\mu + \nu)(\mu + \nu) + 6\mu^2\nu^2(\mu + 2\nu)(\mu + 3\nu)] \\ & + 2\gamma\nu(3\mu + 7\nu)^2 [12\nu^5 + \mu^2(\mu + \nu)(\mu^2 + \nu^2 + \mu\nu)] = 0. \end{aligned} \quad (121)$$

Since these equations are very complicated, we have solved them numerically. Rescaling β , γ , μ and ν as

$$\tilde{\beta} = \beta/|\gamma|, \quad \tilde{\gamma} = \gamma/|\gamma| \quad (= 1 \text{ or } -1), \quad \tilde{\mu} = \mu|\gamma|^{1/6}, \quad \text{and} \quad \tilde{\nu} = \nu|\gamma|^{1/6}, \quad (122)$$

we can always set γ to -1 . We also have to rescale time coordinate as $\tilde{t} = |\gamma|^{-1/6}t$. The typical dynamical time scale is then given by $|\gamma|^{1/6} \sim 0.181818m_{11}^{-1}$, where $m_{11} = \kappa_{11}^{-2/9}$ is the fundamental Planck scale. After this scaling, we have only

one free parameter $\tilde{\beta}$. We find that M-theory ($\tilde{\gamma} = -1$, $\tilde{\beta} = \tilde{\beta}_S = -1/(3 \times 2^5) \approx -0.0104167$) has three solutions

$$\begin{aligned} N_2(\tilde{\mu}_2, \tilde{\nu}_2) &= (0.40731, 0.40731), \\ N_3(\tilde{\mu}_3, \tilde{\nu}_3) &= (0.79683, 0.10793), \\ N_4(\tilde{\mu}_4, \tilde{\nu}_4) &= (0.55570, 0.34253), \end{aligned} \quad (123)$$

and their time-reversed solutions $N'_i(\tilde{\mu}'_i, \tilde{\nu}'_i)$ ($i = 2 \sim 4$) obtained by $(\tilde{\mu}'_i, \tilde{\nu}'_i) = -(\tilde{\mu}_i, \tilde{\nu}_i)$. Other possible solutions for various values of β and γ are given in Ref. 15.

7.2. Stability

Since the solutions obtained above correspond to fixed points in our dynamical system, we can see which solutions are more generic by looking at their stabilities. A linear perturbation analysis around those fixed points was carried out.¹⁵ Setting

$$\frac{du_1}{d\tilde{t}} = \tilde{\mu}_i + A_i e^{\sigma \tilde{t}}, \quad \frac{du_2}{d\tilde{t}} = \tilde{\nu}_i + B_i e^{\sigma \tilde{t}}, \quad (124)$$

where $|A_i|, |B_i| \ll 1$, the perturbation equations are written down. There are five modes ($\sigma = \sigma_a^{(i)}$, $a = 1, 2, \dots, 5$). The M-theory has three solutions (123). Two solutions (N_2 and N_3) have four stable and one unstable modes. The third solution (N_4) has five stable modes, which means that this solution is stable against linear perturbations.

Which of these solutions is most desirable? We want a solution to be rather generic which requires some sort of stability with natural mechanism of ending inflation. This would be achieved if the solution contains many stable as well as tiny unstable modes. Then the spacetime may first approach this solution for wide range of initial conditions and gradually leave it due to the tiny instability, recovering the present Friedmann universe, where we expect the higher order terms become irrelevant. The solution N_3 may give one possible candidate for such a model.

7.3. A Scenario for Large Extra Dimensions

For the interesting solution $N_3(\tilde{\mu}_3, \tilde{\nu}_3) = (0.79683, 0.10793)$ with $\beta = \beta_S$, the scale factor of the external space expands as $e^{0.79683\tilde{t}}$. A successful inflation (resolution of flatness and horizon problems) requires at least 60 e-foldings. We assume that inflation ends after 60 e-foldings, i.e. $0.79683\tilde{t}_{\text{end}} \approx 60$ due to the unstable mode. During inflation the internal space also expands exponentially and its scale becomes $e^{0.10793\tilde{t}_{\text{end}}} \approx 4000$ times larger than the initial scale length, which we assume to be the 11D Planck length (m_{11}^{-1}). The present radius of extra dimensions is then estimated as $R_0 \sim 4000m_{11}^{-1}$. This gives us a model of large extra dimensions.⁸⁴ The 4D Planck mass is given by

$$m_4^2 \sim R_0^7 m_{11}^9 \sim 1.6 \times 10^{25} m_{11}^2. \quad (125)$$

We then find

$$m_{11} \sim 2.5 \times 10^{-13} m_4 \sim 600 \text{TeV}. \quad (126)$$

This is our fundamental energy scale. The present scale of extra dimensions is $4000m_{11}^{-1} \sim 7 \text{ TeV}^{-1}$, which could be observed in the accelerators of next generation.

By putting the argument in a opposite way, we find that this solution gives a natural explanation why the e-folding becomes of the order 60. First suppose that the e-folding of inflation is N . The 3-space expands as $e^N = e^{\tilde{\mu}\tilde{t}_{\text{end}}}$, while the internal space becomes $e^{\tilde{\nu}\tilde{t}_{\text{end}}}$ times larger. It follows from Eq. (125) that

$$m_{11} \sim e^{-\frac{\tilde{\gamma}\nu}{2\mu} N} m_4. \quad (127)$$

Since $m_{11} \gtrsim 1 \text{ TeV}$ from the present experiments, we have a constraint on the e-folding as $N \lesssim 10\mu/\nu$. Then if we have TeV gravity and $\mu \gtrsim 6\nu (> 0)$, this naturally explain why the e-folding of inflation is about 60 and but not so large. Here it is important to note that the solution N_3 with $\gamma < 0$ gives $5.72 < \mu/\nu < 10.22$ (corresponding to $57 \lesssim N \lesssim 102$) for any value of β .

Although the above solution N_3 has one unstable mode, its eigenvalue is of the same order of magnitude as other eigenvalues of stable modes and is a little too large to give enough expansion. It is desirable that the eigenvalue of the unstable mode is much smaller than those of other four stable modes for obtaining large e-folding. The question is if this is possible within this framework of M-theory.

Our starting Lagrangian has ambiguity in the fourth-order correction terms, which are fixed up to the Ricci and scalar curvature tensors. The effect of the ambiguous Ricci and scalar curvature tensors will modify the basic equations. Assuming that this can be taken into account by changing the value of the coefficient $\tilde{\beta}$, such a desirable solution is searched for. An interesting solution is found for $\tilde{\beta} = -0.2025$, for which four modes are stable and the eigenvalue of one unstable mode is very small, i.e. $\sigma_5^{(3)} = 0.01217$. Similar estimate of the size of the extra dimensions then gives again a few TeV, resulting in TeV gravity.

There are several bounds on the allowed size of the extra dimensions from supernova 1987A cooling,^{85,86,87} from graviton decay into diffuse gamma radiation,⁸⁸ or from collider experiments.⁹⁰ All these indicate that the size of the extra dimensions should be greater than a few TeV, which is barely of the same order of magnitude as the above results. So the extra dimensions might be discovered in the next generation of accelerators or further study of astronomical constraints.

If we consider external and internal spaces with nontrivial curvatures, it has been found that there are several more interesting solutions which can give inflationary solutions in the Einstein frame in four dimensions.¹⁶ There not only exact solutions to the basic equations with higher order corrections but also asymptotic past and future solutions are exhausted for generalized de Sitter and power-law solutions. The past asymptotic solutions are useful for describing inflations at early time whereas the future one for understanding accelerating cosmologies at the present time. Cosmological models based on these are under study⁸⁹ and this approach is

expected to lead to further interesting models with large e-foldings for the inflation at early epoch of our universe as well as the present accelerating universe.

Whatever modifications we make to improve some problems or other in these solutions, the emerging common feature of these solutions is that the size of the extra dimensions are large and the value of eleven-dimensional Planck mass is small. What do they mean? They mean that when we make experiments beyond the size of extra dimensions or energy of a few TeV, we begin to see new dimensions open up and strong gravitational effects in higher dimensions set in. There we may have effects of strong gravitational fields and black holes, and have to take full quantum gravity effects into account to understand phenomena at this energy scale.^{91,92} Or we can probe how the quantum gravity behaves there. This is a very interesting possibility.

Although we find a successful exponential expansion and its natural end in this simple setting without introducing any artificial objects into M-theory, this is not enough for a successful inflation. We need a reheating mechanism and have to create a density fluctuation as a seed of cosmic structure. A possible mechanism of reheating is the gravitational particle creation,⁹³ because the background spacetime is time dependent and this might have some oscillation when the internal space settles down to static one which is required to explain our present universe. As for a density perturbation, our model may not give a good scenario because our energy scale is $|\gamma|^{-1/6} \sim 5m_{11} \sim 4$ TeV. We have to use other mechanism for density perturbations such as a curvaton model.⁹⁴ These are the problems left for future study.

8. Other approaches to accelerating universe

The no-go theorem states that no smooth classical compactification of M-theory leads to an accelerating universe. However there are several assumptions in deriving this claim as discussed in Sec. 2.3, so there is a possibility to avoid this theorem if any one of them is violated. The approaches using time-dependent internal spaces and higher order corrections are already discussed. Here we summarize several other attempts in this direction.

It has been argued that Type II* string theories, which are obtained from time-like duality, have Euclidean branes which have de Sitter space in the near-horizon limit.⁹⁵ A possible problem is how to make sense of the Type II* theories because their low-energy effective theories contain higher derivative terms and wrong sign kinetic terms arising from the timelike duality. This is similar to what is called “phantom cosmology.”^{96,97} The recent idea to combine such ghost and higher derivative terms to stabilize a theory could be put in this context.⁹⁸

A compactification of M-theory on a classically singular manifold like a line of finite length is another way to avoid the theorem due to the existence of the singularity in the manifold. By including extra stringy states like wrapped branes, a smooth low-energy action was derived despite the singularity, and it has been

shown that exciting such extra states leads to a period of accelerated expansion,⁵⁴ but it seems difficult to obtain enough e-foldings since the duration of the expansion is typically short.

Asymmetric orientifold is also used to construct de Sitter vacua.⁹⁹ It was argued that it is possible to stabilize all the moduli at a positive extremum of the potential by turning on fluxes in such models. This shares many features with the recent discussions of flux compactification.¹⁰⁰

Many light moduli fields are generated in these string compactifications. The problem is that these fields are difficult to stabilize and tend to be runaway which leads to decompactification to 10 dimensions. In the flux compactification, it is argued that they can be stabilized by turning on fluxes on the compactification manifolds. The argument is that nonperturbative effects generate potentials which stabilize such a runaway at supersymmetric extrema. Similar discussion is given in Ref. 103 using dimensionally reduced theories with scalars and antisymmetric tensors. But this is not enough to get a de Sitter vacuum because all supersymmetric vacua are in anti-de Sitter. We have to break supersymmetry but simple breaking of supersymmetry is not good enough since it will lead to big cosmological constant. It was argued that by adding an anti-D3 brane, supersymmetry can be broken in a controllable way and we can get metastable de Sitter vacuum.^{100,101} However constructing an explicit example of this sort is difficult and no successful scenario has been given. There are lots of efforts to improve this and it is possible that more promising scenario will be given. Possible modifications are proposed by considering Dirac-Born-Infeld action with higher-derivative terms which enforce the slow roll of the scalar fields.¹⁰²

Variants of the higher order corrections discussed above were also considered with negative power of the scalar curvature and/or mixed system with positive powers for the purpose of generating the present accelerating cosmologies and inflation.^{104,105,106} Though this is certainly possible in this modified theory, this modification gives rise to the change of the gravitational law at low curvature and simple models are in conflict with solar system tests of gravity.¹⁰⁷ Ricci curvature squared is also considered with the results of accelerating cosmology.¹⁰⁸ Another common problem with these proposals is that the origin of such terms is not clear.

Some other attempts to derive accelerating cosmologies in higher-dimensional Kaluza-Klein theories includes Refs. 109, 110 with exotic matter. Also systematic methods to find time-dependent solutions in these theories are developed using group theoretic method¹¹¹ or extending the solution space such that the cosmological solutions are geodesic motion in that space,¹¹² and they may be useful to find interesting solutions.

Theories with large number of scalar fields are also considered.¹¹³ This is motivated from the fact that in superstring theories there are a large number of scalar fields. Consistency with cosmic microwave background data suggests that the theory should have some symmetry to avoid excessive isocurvature perturbations. Thus

theories with $SO(N)$ symmetry is studied and it is found that the spectrum of fluctuations are different from single inflaton case.

9. Conclusions

In this paper we have reviewed recent progress in the accelerating cosmologies and inflationary solutions in superstrings or supergravities. The main focus is our time-dependent solutions based on S-brane solutions in M-theory and superstrings with and without higher order corrections, but we have also mentioned several other approaches discussed actively. There are so many directions and literature that we cannot cover all of them. Since the superstring theories or M-theory is supposed to give the quantum theory of gravity, any attempts to give inflation and the present accelerating expansion of our universe should eventually be made in these theories. Then how to avoid the no-go theorem is the main first step to get such solutions.

Our approach using the S-brane solutions is based on the violation of the assumption of time-independence of the size of internal space. We have also shown that the higher order corrections can produce interesting inflationary solutions. The common feature obtained in these solutions is that the size of extra dimensions are rather large of the order of TeV. Future experiments probing this energy scale might show anomalous behaviors related to strong gravitational interactions.

The solutions reviewed here have the virtue that they are the solutions of simple systems without introducing branes or other objects. Any scenario involving such objects would need explanations why such configurations are prepared, but it may be difficult to explain it naturally. The solutions discussed here are natural in the sense not only that they are simply the solutions to the field equations of the fundamental theories without no other input, but also that, as we have shown for solutions in M-theory with fourth-order corrections, general solutions tend to converge the (quasi)stable solutions for wide range of initial conditions. This class of solutions are also interesting in that they have inherent mechanism of ending inflation; they eventually decays due to the tiny instability.

These are the desirable properties, but there also remain some problems (not difficulties) to be studied further. We need a mechanism of keeping the size of the internal space for the S-brane solutions. How to derive reheating and create density fluctuation are the problems that should be studied. We hope that this review may stimulate further study of these approaches to accelerating cosmologies and inflation in the context of superstring theories and M-theory.

Acknowledgements

I would like to thank C. M. Chen, P. M. Ho, I. Neupane, J. E. Wang and K. Maeda for fruitful collaboration and discussions. Thanks are also due to P. K. Townsend and M. N. R. Wohlfarth for discussions. The work was supported in part by the Grant-in-Aid for Scientific Research Fund No. 16540250.

Appendix A. Explicit form of the equations

Here we give general field equations in the theories with higher order corrections for the metric similar to (117), but slightly generalized to p -dimensional external and q -dimensional internal spaces with nontrivial curvatures whose signatures are denoted by σ_p and σ_q , respectively. The action with higher order corrections (108) takes the form¹¹⁴

$$S_{\text{EH}} = \frac{1}{16\pi G} \int d^{p+1}x e^{u_0+pu_1+(q-2)u_2} \left[p_1 \sigma_p e^{2u_2-2u_1} + q_1 \sigma_q - \left\{ p_1 \dot{u}_1^2 + q_1 \dot{u}_2^2 + 2pq \dot{u}_1 \dot{u}_2 \right\} e^{-2u_0+2u_2} \right], \quad (\text{A.1})$$

$$\begin{aligned} S_{\text{GB}} = & \frac{\alpha}{16\pi G} \int d^{p+1}x e^{-u_0+pu_1+(q-2)u_2} \left[p_3 \sigma_p^2 e^{2u_0-4u_1+2u_2} + 2p_1 q_1 \sigma_p \sigma_q e^{2u_0-2u_1} \right. \\ & + q_3 \sigma_q^2 e^{2u_0-2u_2} - 2q_1 \sigma_q \left\{ p_1 \dot{u}_1^2 + 2p(q-2) \dot{u}_1 \dot{u}_2 + (q-2)_3 \dot{u}_2^2 \right\} \\ & - 2p_1 \sigma_p \left\{ (p-2)_3 \dot{u}_1^2 + 2q(p-2) \dot{u}_2 \dot{u}_1 + q_1 \dot{u}_2^2 \right\} e^{2u_2-2u_1} \end{aligned} \quad (\text{A.2})$$

$$- \frac{1}{3} \left\{ p_3 \dot{u}_1^4 + 4qp_2 \dot{u}_1^3 \dot{u}_2 + 6p_1 q_1 \dot{u}_1^2 \dot{u}_2^2 + 4pq_2 \dot{u}_1 \dot{u}_2^3 + q_3 \dot{u}_2^4 \right\} e^{-2u_0+2u_2}, \quad (\text{A.3})$$

$$\begin{aligned} S_4 = & -\frac{\beta/7}{16\pi G} \int d^{1+p}x \sum_{k=0}^8 \frac{8! p! q! \dot{u}_1^{8-k} \dot{u}_2^k e^{-7u_0+pu_1+qu_2}}{k! (8-k)! (p-8+k)! (q-k)!}, \\ S_S = & \frac{1}{16\pi G} \int d^{1+p}x \gamma e^{-7u_0+pu_1+qu_2} \left[p_1 X^2 \{X + 2(\dot{u}_1^2 + \sigma_p e^{2u_0-2u_1})\}^2 \right. \\ & + q_1 Y^2 \{Y + 2(\dot{u}_2^2 + \sigma_q e^{2u_0-2u_2})\}^2 + 2pq \{XY + (X+Y)\dot{u}_1 \dot{u}_2\}^2 \\ & + 3p_2 (\dot{u}_1^2 + \sigma_p e^{2u_0-2u_1})^4 + 3q_2 (\dot{u}_2^2 + \sigma_q e^{2u_0-2u_2})^4 \\ & + p_1 q \{\dot{u}_1 \dot{u}_2 + 2(\dot{u}_1^2 + \sigma_p e^{2u_0-2u_1})\}^2 \dot{u}_1^2 \dot{u}_2^2 \\ & \left. + pq_1 \{\dot{u}_1 \dot{u}_2 + 2(\dot{u}_2^2 + \sigma_q e^{2u_0-2u_2})\}^2 \dot{u}_1^2 \dot{u}_2^2 \right], \quad (\text{A.4}) \end{aligned}$$

where S_4 is given only for $\sigma_p = \sigma_q = 0$ case but others are for general case, and we use the following notation throughout this paper:

$$\begin{aligned} (p-m)_n &\equiv (p-m)(p-m-1)(p-m-2) \cdots (p-n), \\ (q-m)_n &\equiv (q-m)(q-m-1)(q-m-2) \cdots (q-n), \\ A_p &\equiv \dot{u}_1^2 + \sigma_p e^{2(u_0-u_1)}, \quad A_q \equiv \dot{u}_2^2 + \sigma_q e^{2(u_0-u_2)}, \\ X &\equiv \ddot{u}_1 - \dot{u}_0 \dot{u}_1 + \dot{u}_1^2, \quad Y \equiv \ddot{u}_2 - \dot{u}_0 \dot{u}_2 + \dot{u}_2^2. \end{aligned} \quad (\text{A.5})$$

From the variation of the total action (108) with respect to u_0, u_1 and u_2 , we find three basic field equations for general case with nonvanishing curvature:¹⁶

$$F \equiv \sum_{n=1}^4 F_n + F_S = 0, \quad (\text{A.6})$$

$$F^{(p)} \equiv \sum_{n=1}^4 f_n^{(p)} + \left(\sum_{n=1}^4 g_n^{(p)} \right) X + \left(\sum_{n=1}^4 h_n^{(p)} \right) Y + F_S^{(p)} = 0, \quad (\text{A.7})$$

$$F^{(q)} \equiv \sum_{n=1}^4 f_n^{(q)} + \left(\sum_{n=1}^4 g_n^{(q)} \right) Y + \left(\sum_{n=1}^4 h_n^{(q)} \right) X + F_S^{(q)} = 0, \quad (\text{A.8})$$

where each terms are summarized below.

(1) EH action

$$F_1 = e^{-u_0} [p_1 A_p + q_1 A_q + 2pq\dot{u}_1\dot{u}_2], \quad (\text{A.9})$$

$$f_1^{(p)} = e^{-u_0} [(p-1)_2 A_p + q_1 A_q + 2(p-1)q\dot{u}_1\dot{u}_2], \quad (\text{A.10})$$

$$f_1^{(q)} = e^{-u_0} [p_1 A_p + (q-1)_2 A_q + 2p(q-1)\dot{u}_1\dot{u}_2], \quad (\text{A.11})$$

$$g_1^{(p)} = 2(p-1)e^{-u_0}, \quad (\text{A.12})$$

$$g_1^{(q)} = 2(q-1)e^{-u_0}, \quad (\text{A.13})$$

$$h_1^{(p)} = 2qe^{-u_0}, \quad (\text{A.14})$$

$$h_1^{(q)} = 2pe^{-u_0}. \quad (\text{A.15})$$

(2) GB action

$$\begin{aligned} F_2 = \alpha e^{-3u_0} & [p_3 A_p^2 + 2p_1 q_1 A_p A_q + q_3 A_q^2 + 4\dot{u}_1\dot{u}_2 (p_2 q A_p + p q_2 A_q) \\ & + 4p_1 q_1 \dot{u}_1^2 \dot{u}_2^2], \end{aligned} \quad (\text{A.16})$$

$$\begin{aligned} f_2^{(p)} = \alpha e^{-3u_0} & [(p-1)_4 A_p^2 + 2(p-1)_2 q_1 A_p A_q + q_3 A_q^2 \\ & + 4\dot{u}_1\dot{u}_2 ((p-1)_3 q A_p + (p-1)_2 q_2 A_q) + 4(p-1)_2 q_1 \dot{u}_1^2 \dot{u}_2^2], \end{aligned} \quad (\text{A.17})$$

$$\begin{aligned} f_2^{(q)} = \alpha e^{-3u_0} & [(q-1)_4 A_q^2 + 2(q-1)_2 p_1 A_p A_q + p_3 A_p^2 \\ & + 4\dot{u}_1\dot{u}_2 ((q-1)_3 p A_q + (q-1)_2 p_2 A_p) + 4(q-1)_2 p_1 \dot{u}_1^2 \dot{u}_2^2], \end{aligned} \quad (\text{A.18})$$

$$g_2^{(p)} = 4(p-1)\alpha e^{-3u_0} [(p-2)_3 A_p + q_1 A_q + 2(p-2)q\dot{u}_1\dot{u}_2], \quad (\text{A.19})$$

$$g_2^{(q)} = 4(q-1)\alpha e^{-3u_0} [(q-2)_3 A_q + p_1 A_p + 2(q-2)p\dot{u}_1\dot{u}_2], \quad (\text{A.20})$$

$$h_2^{(p)} = 4q\alpha e^{-3u_0} [(p-1)_2 A_p + (q-1)_2 A_q + 2(p-1)(q-1)\dot{u}_1\dot{u}_2], \quad (\text{A.21})$$

$$h_2^{(q)} = 4p\alpha e^{-3u_0} [(q-1)_2 A_q + (p-1)_2 A_p + 2(p-1)(q-1)\dot{u}_1\dot{u}_2]. \quad (\text{A.22})$$

(3) Lovelock action

$$\begin{aligned} F_4 = \beta e^{-7u_0} & [p_7 A_p^4 + 4p_5 q_1 A_p^3 A_q + 6p_3 q_3 A_p^2 A_q^2 + 4p_1 q_5 A_p A_q^3 + q_7 A_q^4 \\ & + 8\dot{u}_1\dot{u}_2 (p_6 q A_p^3 + 3p_4 q_2 A_p^2 A_q + 3p_2 q_4 A_p A_q^2 + p q_6 A_q^3) + 24\dot{u}_1^2 \dot{u}_2^2 (p_5 q_1 A_p^2 \\ & + 2p_3 q_3 A_p A_q + p_1 q_5 A_q^2) + 32\dot{u}_1^3 \dot{u}_2^3 (p_4 q_2 A_p + p_2 q_4 A_q) + 16p_3 q_3 \dot{u}_1^4 \dot{u}_2^4], \end{aligned} \quad (\text{A.23})$$

$$\begin{aligned} f_4^{(p)} = \beta e^{-7u_0} & [(p-1)_8 A_p^4 + 4(p-1)_6 q_1 A_p^3 A_q + 6(p-1)_4 q_3 A_p^2 A_q^2 \\ & + 4(p-1)_2 q_5 A_p A_q^3 + q_7 A_q^4 + 8\dot{u}_1\dot{u}_2 ((p-1)_7 q A_p^3 + 3(p-1)_5 q_2 A_p^2 A_q \\ & + 3(p-1)_3 q_4 A_p A_q^2 + (p-1)_2 q_6 A_q^3) + 24\dot{u}_1^2 \dot{u}_2^2 ((p-1)_6 q_1 A_p^2 \\ & + 2(p-1)_4 q_3 A_p A_q + (p-1)_2 q_5 A_q^2) \\ & + 32\dot{u}_1^3 \dot{u}_2^3 ((p-1)_5 q_2 A_p + (p-1)_3 q_4 A_q) + 16(p-1)_4 q_3 \dot{u}_1^4 \dot{u}_2^4], \end{aligned} \quad (\text{A.24})$$

$$\begin{aligned} f_4^{(q)} = \beta e^{-7u_0} & [(q-1)_8 A_q^4 + 4(q-1)_6 p_1 A_q^3 A_p + 6(q-1)_4 p_3 A_p^2 A_q^2 \\ & + 4(q-1)_2 p_5 A_q A_p^3 + p_7 A_p^4 + 8\dot{u}_1\dot{u}_2 ((q-1)_7 p A_q^3 + 3(q-1)_5 p_2 A_q^2 A_p \end{aligned}$$

$$\begin{aligned}
& +3(q-1)_3p_4A_qA_p^2+(q-1)p_6A_p^3)+24\dot{u}_1^2\dot{u}_2^2((q-1)_6p_1A_q^2 \\
& +2(q-1)_4p_3A_pA_q+(q-1)_2p_5A_p^2) \\
& +32\dot{u}_1^3\dot{u}_2^3((q-1)_5p_2A_q+(q-1)_3p_4A_p)+16(q-1)_4p_3\dot{u}_1^4\dot{u}_2^4], \quad (\text{A.25})
\end{aligned}$$

$$\begin{aligned}
g_4^{(p)} = & 8(p-1)\beta e^{-7u_0}[(p-2)_7A_p^3+3(p-2)_5q_1A_p^2A_q+3(p-2)_3q_3A_pA_q^2+q_5A_q^3 \\
& +6\dot{u}_1\dot{u}_2((p-2)_6qA_p^2+2(p-2)_4q_2A_pA_q+(p-2)_2q_4A_q^2) \\
& +12\dot{u}_1^2\dot{u}_2^2((p-2)_5q_1A_p+(p-2)_3q_3A_q)+8(p-2)_4q_2\dot{u}_1^3\dot{u}_2^3], \quad (\text{A.26})
\end{aligned}$$

$$\begin{aligned}
g_4^{(q)} = & 8(q-1)\beta e^{-7u_0}[(q-2)_7A_q^3+3(q-2)_5p_1A_q^2A_p+3(q-2)_3p_3A_qA_p^2+p_5A_p^3 \\
& +6\dot{u}_1\dot{u}_2((q-2)_6pA_q^2+2(q-2)_4p_2A_pA_q+(q-2)_2p_4A_p^2) \\
& +12\dot{u}_1^2\dot{u}_2^2((q-2)_5p_1A_q+(q-2)_3p_3A_p)+8(q-2)_4p_2\dot{u}_1^3\dot{u}_2^3], \quad (\text{A.27})
\end{aligned}$$

$$\begin{aligned}
h_4^{(p)} = & 8q\beta e^{-7u_0}[(p-1)_6A_p^3+3(p-1)_4(q-1)_2A_p^2A_q+3(p-1)_2(q-1)_4A_pA_q^2 \\
& +(q-1)_6A_q^3+6\dot{u}_1\dot{u}_2((p-1)_5(q-1)_1A_p^2+2(p-1)_3(q-1)_3A_pA_q \\
& +(p-1)(q-1)_5A_q^2)+12\dot{u}_1^2\dot{u}_2^2((p-1)_4(q-1)_2A_p+(p-1)_2(q-1)_4A_q) \\
& +8(p-1)_3(q-1)_3\dot{u}_1^3\dot{u}_2^3], \quad (\text{A.28})
\end{aligned}$$

$$\begin{aligned}
h_4^{(q)} = & 8p\beta e^{-7u_0}[(q-1)_6A_q^3+3(q-1)_4(p-1)_2A_q^2A_p+3(q-1)_2(p-1)_4A_qA_p^2 \\
& +(p-1)_6A_p^3+6\dot{u}_1\dot{u}_2((q-1)_5(p-1)_1A_q^2+2(p-1)_3(q-1)_3A_pA_q \\
& +(q-1)(p-1)_5A_p^2)+12\dot{u}_1^2\dot{u}_2^2((q-1)_4(p-1)_2A_q+(q-1)_2(p-1)_4A_p) \\
& +8(p-1)_3(q-1)_3\dot{u}_1^3\dot{u}_2^3]. \quad (\text{A.29})
\end{aligned}$$

(4) S_S action

$$\begin{aligned}
F_S = & \gamma e^{-pu_1-qu_2} \left[-7L_4 + 2\sigma_p e^{2(u_0-u_1)} \frac{\partial L_4}{\partial A_p} + 2\sigma_q e^{2(u_0-u_2)} \frac{\partial L_4}{\partial A_q} \right. \\
& \left. + \frac{d}{dt} \left(\dot{u}_1 \frac{\partial L_4}{\partial X} + \dot{u}_2 \frac{\partial L_4}{\partial Y} \right) \right], \quad (\text{A.30})
\end{aligned}$$

$$\begin{aligned}
pF_S^{(p)} = & \gamma e^{-pu_1-qu_2} \left[pL_4 - 2\sigma_p e^{2(u_0-u_1)} \frac{\partial L_4}{\partial A_p} \right. \\
& \left. + \frac{d}{dt} \left((\dot{u}_0 - 2\dot{u}_1) \frac{\partial L_4}{\partial X} - 2\dot{u}_1 \frac{\partial L_4}{\partial A_p} - \frac{\partial L_4}{\partial \dot{u}_1} \right) + \frac{d^2}{dt^2} \left(\frac{\partial L_4}{\partial X} \right) \right] \quad (\text{A.31})
\end{aligned}$$

$$\begin{aligned}
qF_S^{(q)} = & \gamma e^{-pu_1-qu_2} \left[qL_4 - 2\sigma_q e^{2(u_0-u_2)} \frac{\partial L_4}{\partial A_q} \right. \\
& \left. + \frac{d}{dt} \left((\dot{u}_0 - 2\dot{u}_2) \frac{\partial L_4}{\partial Y} - 2\dot{u}_2 \frac{\partial L_4}{\partial A_q} - \frac{\partial L_4}{\partial \dot{u}_2} \right) + \frac{d^2}{dt^2} \left(\frac{\partial L_4}{\partial Y} \right) \right], \quad (\text{A.32})
\end{aligned}$$

where

$$\begin{aligned}
L_4 = & e^{-7u_0+pu_1+qu_2} [p_1X^2(X+2A_p)^2+q_1Y^2(Y+2A_q)^2+3p_2A_p^4+3q_2A_q^4 \\
& +2pq(XY+(X+Y)\dot{u}_1\dot{u}_2)^2+p_1q\dot{u}_1^2\dot{u}_2^2(\dot{u}_1\dot{u}_2+2A_p)^2 \\
& +pq_1\dot{u}_1^2\dot{u}_2^2(\dot{u}_1\dot{u}_2+2A_q)^2], \quad (\text{A.33})
\end{aligned}$$

$$\frac{\partial L_4}{\partial X} = 4pe^{-7u_0+pu_1+qu_2} [(p-1)X(X+A_p)(X+2A_p)$$

$$+ q(Y + \dot{u}_1 \dot{u}_2)(XY + (X + Y)\dot{u}_1 \dot{u}_2)], \quad (\text{A.34})$$

$$\begin{aligned} \frac{\partial L_4}{\partial Y} = 4qe^{-7u_0+pu_1+qu_2} & [(q-1)Y(Y+A_q)(Y+2A_q) \\ & + p(X+\dot{u}_1 \dot{u}_2)(XY + (X + Y)\dot{u}_1 \dot{u}_2)], \end{aligned} \quad (\text{A.35})$$

$$\begin{aligned} \frac{\partial L_4}{\partial A_p} = 4p_1e^{-7u_0+pu_1+qu_2} & [X^2(X+2A_p) + 3(p-2)A_p^3 \\ & + qu_1^2 u_2^2(\dot{u}_1 \dot{u}_2 + 2A_p)], \end{aligned} \quad (\text{A.36})$$

$$\begin{aligned} \frac{\partial L_4}{\partial A_q} = 4q_1e^{-7u_0+pu_1+qu_2} & [Y^2(Y+2A_q) + 3(q-2)A_q^3 \\ & + pu_1^2 u_2^2(\dot{u}_1 \dot{u}_2 + 2A_q)], \end{aligned} \quad (\text{A.37})$$

$$\begin{aligned} \frac{\partial L_4}{\partial \dot{u}_1} = 4pq e^{-7u_0+pu_1+qu_2} \dot{u}_2 & [(X+Y)(XY + (X + Y)\dot{u}_1 \dot{u}_2) \\ & + (p-1)\dot{u}_1 \dot{u}_2(\dot{u}_1 \dot{u}_2 + A_p)(\dot{u}_1 \dot{u}_2 + 2A_p) \\ & + (q-1)\dot{u}_1 \dot{u}_2(\dot{u}_1 \dot{u}_2 + A_q)(\dot{u}_1 \dot{u}_2 + 2A_q)], \end{aligned} \quad (\text{A.38})$$

$$\begin{aligned} \frac{\partial L_4}{\partial \dot{u}_2} = 4pq e^{-7u_0+pu_1+qu_2} \dot{u}_1 & [(X+Y)(XY + (X + Y)\dot{u}_1 \dot{u}_2) \\ & + (p-1)\dot{u}_1 \dot{u}_2(\dot{u}_1 \dot{u}_2 + A_p)(\dot{u}_1 \dot{u}_2 + 2A_p) \\ & + (q-1)\dot{u}_1 \dot{u}_2(\dot{u}_1 \dot{u}_2 + A_q)(\dot{u}_1 \dot{u}_2 + 2A_q)]. \end{aligned} \quad (\text{A.39})$$

Since u_0 is a gauge freedom of time coordinate, we have three equations (A.6) – (A.8) for two variables u_1 and u_2 . It looks like an over-determinant system. However, these three equations are not independent since the following relation is valid:

$$\dot{F} + (p\dot{u}_1 + q\dot{u}_2 - \dot{u}_0)F = p\dot{u}_1 F^{(p)} + q\dot{u}_2 F^{(q)}. \quad (\text{A.40})$$

If $F = 0$ and $F^{(p)} = 0$ (or $F^{(q)} = 0$), we obtain $\dot{u}_2 F^{(q)} = 0$ (or $\dot{u}_1 F^{(p)} = 0$), since $F = 0$ is a constraint equation and its time derivative also vanishes. However, if we have only $F^{(p)} = 0$ and $F^{(q)} = 0$, $F = 0$ is not necessarily obeyed. As mentioned above, this is because $F = 0$ is a constraint equation, while $F^{(p)} = 0$ and $F^{(q)} = 0$ are dynamical equations. To satisfy the constraint equation, we must impose the initial condition. Consequently, we can solve the two equations $F = 0$ and $F^{(p)} = 0$ (or $F^{(q)} = 0$) instead of trying to solve all three equations.

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